حمل الآن

مجانا وحصريا

امتحانات رقم (1)

الترم الثانى

ذاكرولى
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1 Cairo Governorate

Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer from those given :

1. The inscribed angle drawn in a semicircle is .... angle.
   (a) a straight (b) an obtuse (c) a right (d) an acute
2. The circumference of the circle whose radius length is r cm. equals .... cm.
   (a) $2 \pi r$ (b) πr (c) πr^2 (d) $2 \pi r^2$
3. The sum of measures of the interior angles of the quadrilateral equals ....
   (a) 90° (b) 180° (c) 270° (d) 360°
4. The area of the square whose side length is 3 cm. equals .... cm^2.
   (a) 6 (b) 9 (c) 12 (d) 18
5. We can not draw a circle passes through the vertices of the ....
   (a) square. (b) parallelogram. (c) rectangle. (d) isosceles trapezium.
6. A circle M of radius length 6 cm. , A is a point in the plane if MA = 3 cm.
   , then A lies .... the circle.
   (a) inside (b) outside (c) on (d) at the centre of

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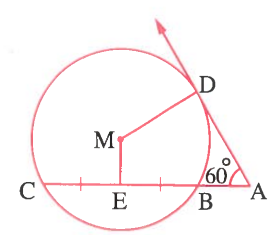
2 [a] In the opposite figure :



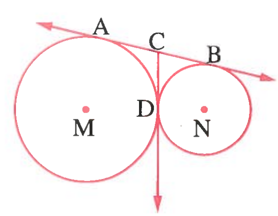
$\overrightarrow{AD}$ is a tangent to the circle M at D

, $\overline{AC}$ intersects the circle M at B , C

, m (∠ A) = 60° , E is the midpoint of $\overline{BC}$

Find with prove : m (∠ DME)

[b] In the opposite figure :



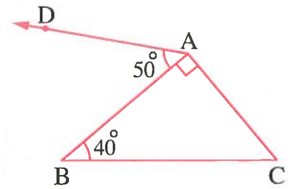
M and N are two circles touching externally at D

, $\overleftrightarrow{AB}$ is a common tangent to them at A , B

, $\overrightarrow{CD}$ is a common tangent to them at D

Prove that : C is the midpoint of $\overline{AB}$

3 [a] **In the opposite figure :**



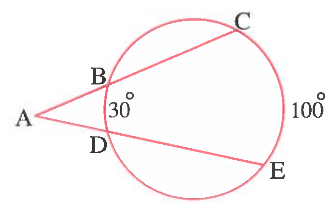
m (∠ DAB) = 50° , $\overline{BA} \perp \overline{AC}$

, m (∠ ABC) = 40°

Prove that : $\overrightarrow{AD}$ is a tangent to the circle

that passes through the vertices of the triangle ABC

[b] **In the opposite figure :**



m ($\overset{\frown}{EC}$) = 100° , m ($\overset{\frown}{BD}$) = 30°

, **find with proof :** m (∠ A)

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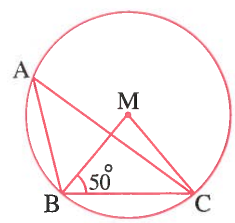
4 [a] **In the opposite figure :**



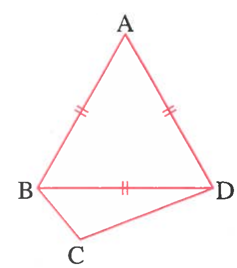
A circle of centre M

, m (∠ MBC) = 50°

Find with proof : 1 m (∠ BMC)

2 m (∠ BAC)

[b] **In the opposite figure :**



ABCD is a cyclic quadrilateral

, Δ ABD is an equilateral triangle

Find with proof : m (∠ C)

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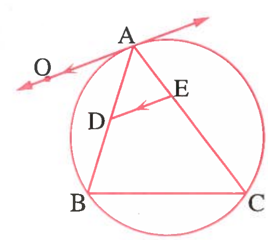
5 [a] **In the opposite figure :**



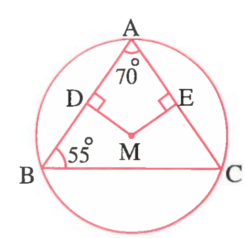
$\overleftrightarrow{AO}$ is a tangent to the circle at A

, $\overleftrightarrow{AO}$ // $\overline{ED}$

Prove that :

The figure DBCE is a cyclic quadrilateral

[b] **In the opposite figure :**



A circle of centre M , $\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

, m (∠ A) = 70° , m (∠ B) = 55°

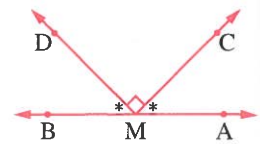
Prove that : MD = ME

2 Giza Governorate

Answer the following questions :

1 Choose the correct answer :

1 In the opposite figure :



$M \in \overleftrightarrow{AB}$, $\overrightarrow{MC} \perp \overrightarrow{MD}$, m (∠ AMC) = m (∠ BMD)

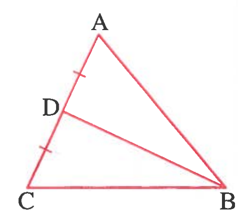
, then m (∠ AMC) = ...

(a) 90° (b) 135° (c) 60° (d) 45°

2 In Δ ABC , if $(AB)^2 > (AC)^2 + (BC)^2$, then the type of ∠ C is ....

(a) obtuse. (b) acute. (c) straight. (d) right.

3 In the opposite figure :



If ABC is a triangle , D is a midpoint of $\overline{AC}$

if the area of Δ ABD = 20 cm^2.

, then the area of Δ ABC = ... cm^2.

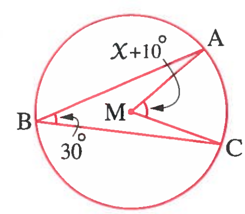
(a) 10 (b) 20

(c) 40 (d) 30

4 If the straight line L is a tangent to the circle whose diameter length is 10 cm. , then the distance between L and centre of the circle is .... cm.

(a) 4 (b) 5 (c) 6 (d) 7

5 In the opposite figure :



M is a circle , if m (∠ AMC) = (X + 10°) , m (∠ B) = 30°

, then the value of X is ....

(a) 130° (b) 100°

(c) 50° (d) 40°

6 If the two circles M and N are touching internally , their radii lengths are 3 cm. and 5 cm. , then MN = ... cm.

(a) 1 (b) 2 (c) 3 (d) 8

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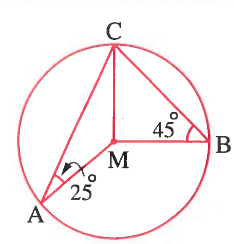
2 [a] In the opposite figure :



M is a circle , m (∠ MBC) = 45°

, m (∠ MAC) = 25°

, **find :** m (∠ AMB)

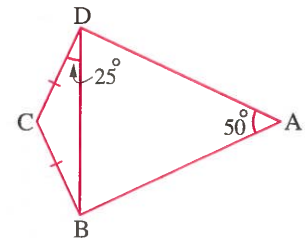
[b] In the opposite figure :

$CD = CB$, $m(\angle CDB) = 25°$

, $m(\angle A) = 50°$

Prove that :

ABCD is a cyclic quadrilateral.



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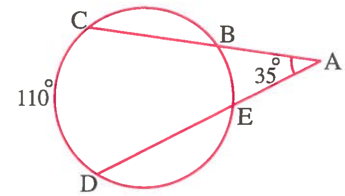
3 **[a] In the opposite figure :**

If $\overrightarrow{CB} \cap \overrightarrow{DE} = \{A\}$, $m(\angle A) = 35°$

, $m(\overset{\frown}{CD}) = 110°$

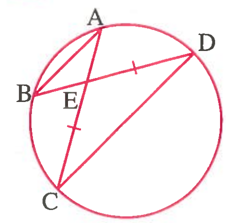
, then find : $m(\overset{\frown}{BE})$



[b] In the opposite figure :

If $\overline{AC} \cap \overline{BD} = \{E\}$, $ED = EC$

, then prove that : $AE = BE$



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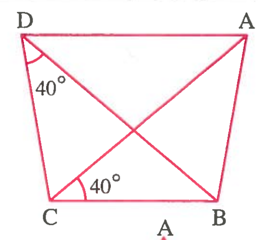
4 **[a] In the opposite figure :**

If ABCD is a cyclic quadrilateral

, $m(\angle BDC) = m(\angle ACB) = 40°$

, then prove that : Δ ABC is isosceles



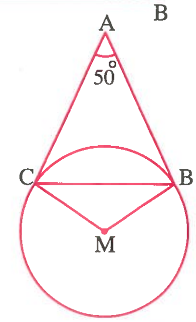
[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two tangent-segments

to the circle M at B , C , $m(\angle A) = 50°$

Find : 1 $m(\angle ABC)$

2 $m(\angle MCB)$



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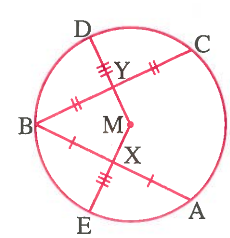
5 **[a] In the opposite figure :**

If $\overline{AB}$, $\overline{BC}$ two chords in circle M

, X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{BC}$ and $XE = YD$

Prove that : $AB = BC$

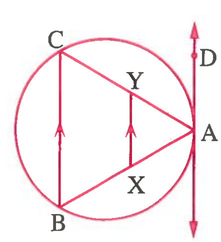


[b] In the opposite figure :

If $\overleftrightarrow{AD}$ is a tangent to the circle at A

, $X \in \overline{AB}$, $Y \in \overline{AC}$ and $\overline{XY} \,//\, \overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle which passes through the vertices of Δ AXY



3 Alexandria Governorate

Answer the following questions : *(Calculators are permitted)*

1 Choose the correct answer from those given :

1 M and N are two circles , the lengthes of their radii are 9 cm. , 4 cm. and MN = 5 cm. , then the two circles are ....

(a) distant. (b) intersecting.

(c) touching internally. (d) touching externally.

2 The number of axes of symmetry of a square is ....

(a) 1 (b) 3 (c) 2 (d) 4

3 The measure of the inscribed angle equals .... the measure of opposite arc.

(a) half (b) third (c) quarter (d) twice

4 The area of the circle whose diameter length is 14 cm. equals .... cm^2.

(a) 14π (b) 49π (c) 7π (d) 28π

5 ABC is a triangle , AB = AC , m (∠ A) = 80° , then m (∠ C) = ...

(a) 80° (b) 25° (c) 50° (d) 100°

6 ABCD is a cyclic quadrilateral , if m (∠ A) = 3 m (∠ C) , then m (∠ A) = ...

(a) 45° (b) 90° (c) 120° (d) 135°

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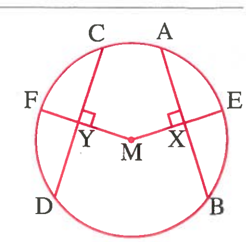
2 [a] In the opposite figure :

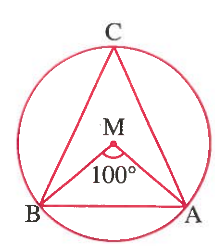


AB = DC , $\overline{ME} \perp \overline{AB}$

, $\overline{MF} \perp \overline{CD}$

Prove that : XE = YF

[b] In the opposite figure :



M is a circle , m (∠ BMA) = 100°

Find : 1 m (∠ ACB)

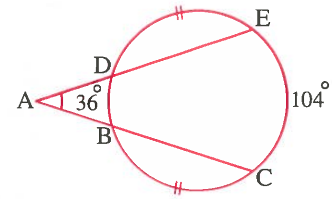
2 m (∠ MAB)

3 [a] **In the opposite figure :**

$m(\angle A) = 36°$, $m(\overset{\frown}{EC}) = 104°$

, $m(\overset{\frown}{DE}) = m(\overset{\frown}{BC})$

Find : 1 $m(\overset{\frown}{BD})$ 2 $m(\overset{\frown}{DE})$

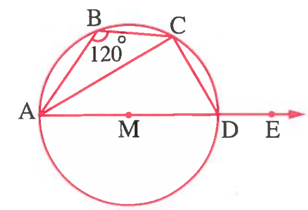


[b] **In the opposite figure :**

ABCD is quadrilateral , $\overline{AD}$ is a diameter of the circle M

, $E \in \overrightarrow{AD}$, $m(\angle B) = 120°$

Find : 1 $m(\angle CDE)$ 2 $m(\angle CAD)$



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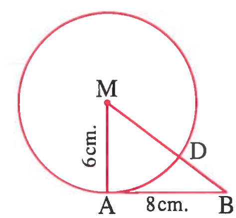
4 [a] **In the opposite figure :**

$\overline{AB}$ is a tangent-segment to the circle M at A

, AM = 6 cm. , AB = 8 cm.

Find by proof the length of $\overline{BD}$

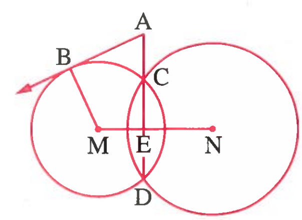


[b] **In the opposite figure :**

M and N are two intersecting circles at C and D

, $\overrightarrow{AB}$ is tangent of the circle M at B , $\overline{MN} \cap \overline{CD} = \{E\}$

Prove that : ABME is a cyclic quadrilateral.



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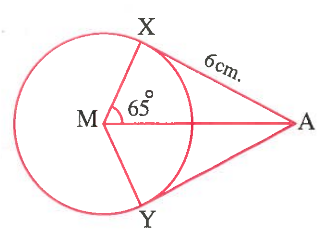
5 [a] **In the opposite figure :**

$\overline{AX}$, $\overline{AY}$ are two tangent-segments to the circle M at X , Y respectively

, $m(\angle AMX) = 65°$, AX = 6 cm.

Find by proof : 1 The length of $\overline{AY}$

2 $m(\angle XAY)$



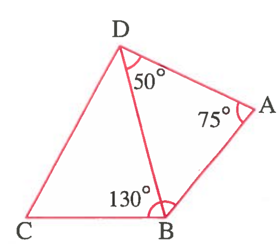
[b] **In the opposite figure :**

ABCD is a cyclic quadrilateral

, $m(\angle ADB) = 50°$, $m(\angle A) = 75°$

, $m(\angle ABC) = 130°$

Prove that : $\overline{BC}$ is a tangent-segment to the circle which passes through the points A , B and D



4 El-Kalyoubia Governorate

Answer the following questions :

1 Choose the correct answer from the given ones :

1 If Δ ABC is right at B , then m ($\angle$ A) + m ($\angle$ C) = ...

(a) 30° (b) 45° (c) 60° (d) 90°

2 If $\overline{AB}$ is a diameter in the circle M , $\overleftrightarrow{AC}$ and $\overleftrightarrow{BD}$ are two tangents to the circle , then $\overleftrightarrow{AC}$ is .... $\overleftrightarrow{BD}$

(a) intersecting (b) parallel to (c) perpendicular to (d) coincident to

3 The two similar polygons their corresponding angles are ....

(a) congruent. (b) different. (c) supplementary. (d) complementary.

4 If $\overleftrightarrow{AB} \cap$ the circle M = $\{A , B\}$, then $\overleftrightarrow{AB} \cap$ the surface of the circle M = ...

(a) $\{A , B\}$ (b) $\overline{AB}$ (c) $\overrightarrow{AB}$ (d) $\overrightarrow{BA}$

5 If Δ ABC has one axis of symmetry and its side lengths are 10 cm. , 5 cm. , x cm. , then x = ... cm.

(a) 5 (b) 8 (c) 10 (d) 12

6 If ABCD is a quadrilateral , m ($\angle$ A) + m ($\angle$ C) = 180° , then ABCD is ....

(a) a cyclic quadrilateral. (b) a rhombus.

(c) a parallelogram. (d) a trapezium.

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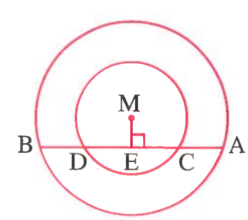
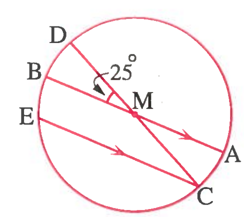
2 [a] In the opposite figure :



Two concentric circles their centre is M , $\overline{AB}$ is chord in the greater circle and intersects the smaller circle in C and D , $\overline{ME} \perp \overline{AB}$

Prove that : AC = BD

[b] In the opposite figure :

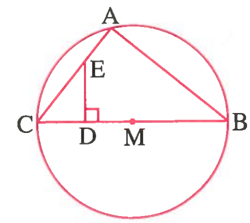


$\overline{AB}$ and $\overline{CD}$ are two diameters in the circle M , where m ($\angle$ DMB) = 25° , $\overline{CE}$ // $\overline{AB}$

Find : m $(\overset{\frown}{BE})$

3 **[a]** By using the geometric instruments , draw $\overline{AB}$ of length 4 cm. , then draw a circle passes through the two points A and B and its radius length 3 cm. , what is the number of solutions ?

[b] In the opposite figure :



$\overline{BC}$ is a diameter in the circle M

, $\overline{ED} \perp \overline{BC}$

Prove that : 1 ABDE is a cyclic quadrilateral.

2 $m(\angle CED) = \frac{1}{2} m(\overset{\frown}{AC})$

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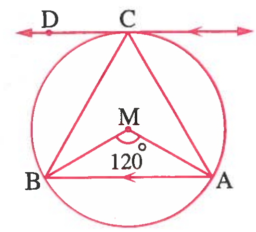
4 **[a] In the opposite figure :**

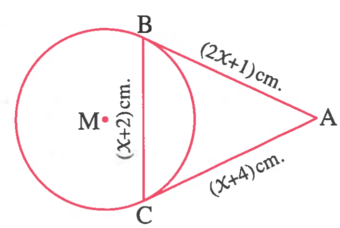


The circle M passes through the vertices of Δ ABC , $m(\angle AMB) = 120°$

, $\overleftrightarrow{CD}$ is a tangent to the circle M at C , $\overleftrightarrow{CD} \parallel \overline{AB}$

, **prove that :** The Δ ABC is an equilateral triangle.

[b] In the opposite figure :



$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M at B and C respectively , $AB = (2X + 1)$ cm.

, $AC = (X + 4)$ cm. , $BC = (X + 2)$ cm.

1 **Find :** the value of X

2 **Calculate :** the perimeter of Δ ABC

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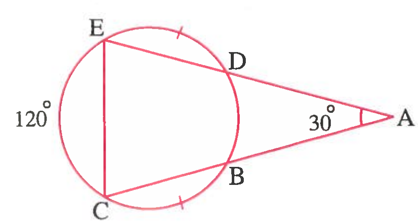
5 **[a] In the opposite figure :**



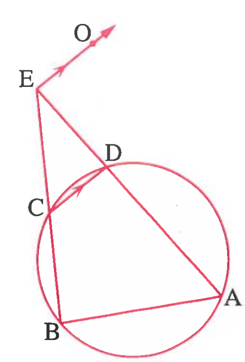
$\overrightarrow{ED} \cap \overrightarrow{CB} = \{A\}$, $m(\overset{\frown}{DE}) = m(\overset{\frown}{BC})$

, $m(\angle A) = 30°$, $m(\overset{\frown}{CE}) = 120°$

1 **Find :** $m(\overset{\frown}{BD})$

2 **Prove that :** AE = AC

[b] In the opposite figure :



ABCD is a cyclic quadrilateral

, $\overrightarrow{AD} \cap \overrightarrow{BC} = \{E\}$, $\overrightarrow{EO} \parallel \overline{CD}$

Prove that : $\overrightarrow{EO}$ is a tangent to the circle which passes through the vertices of Δ EBA

5 El-Sharkia Governorate

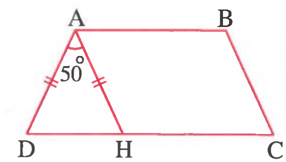
Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer from those given :

1 M and N are two circles touching externally , their radii lengths are 7 cm. and 3 cm. , then the length of $\overline{MN}$ = ... cm.

(a) 8 (b) 4 (c) 2 (d) 10

2 **In the opposite figure :**



If ABCD is a cyclic quadrilateral

, AH = AD , m (∠ HAD) = 50°

, then m (∠ B) = ...

(a) 115° (b) 65° (c) 130° (d) 105°

3 If AB = 6 cm. , then the number of circles which pass through the two points A , B and their radii of length 6 cm. equals ....

(a) zero (b) 1 (c) 2 (d) 6

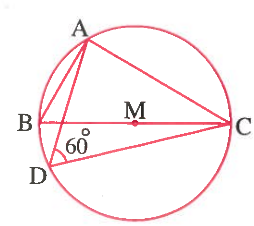
4 The length of the arc which subtendes a central angle of measure 120° in a circle of diameter length 42 cm. equals .... cm.

(a) 28 (b) 22 (c) 21 (d) 44

5 The number of axes of symmetry of a semicircle is ....

(a) zero (b) 1 (c) 2 (d) infinite.

6 **In the opposite figure :**



$\overline{BC}$ is a diameter of the circle M , m (∠ D) = 60°

, AB = 5 cm. , then the area of the circle M = ... π cm^2.

(a) 25 (b) 12

(c) 100 (d) 49

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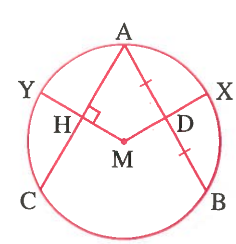
2 [a] **In the opposite figure :**



M is a circle , D is the midpoint of $\overline{AB}$

, $\overline{MH} \perp \overline{AC}$, AB = AC

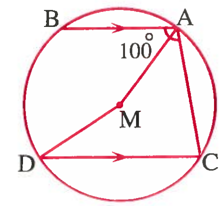
Prove that : DX = HY

[b] In the opposite figure :

M is a circle where $\overline{AB} \parallel \overline{CD}$

, m (∠ CAB) = 100°

Find : m (∠ AMD)

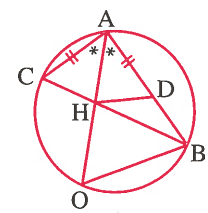


3 **[a] In the opposite figure :**

AD = AC , $\overrightarrow{AO}$ bisects ∠ CAB

Prove that :

The figure HDBO is a cyclic quadrilateral.

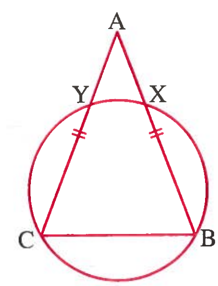


[b] In the opposite figure :

$\overline{BX}$, $\overline{CY}$ are two chords equal in length

where $\overrightarrow{BX} \cap \overrightarrow{CY} = \{A\}$

Prove that : AX = AY



4 **[a] In the opposite figure :**

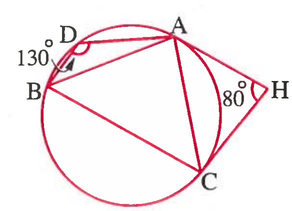
$\overline{HA}$, $\overline{HC}$ are two tangent-segments

of the circle at A , C where m (∠ AHC) = 80°

, m (∠ ADB) = 130°

Prove that : 1 AB = AC

2 $\overrightarrow{AB}$ is a tangent of the circle which passes through the points A , C and H

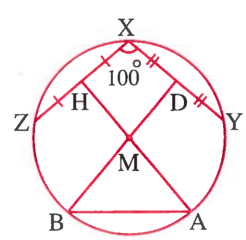


[b] In the opposite figure :

M is a circle where m (∠ YXZ) = 100°

, D is the midpoint of $\overline{XY}$, H is the midpoint of $\overline{XZ}$

Prove that : AB > r



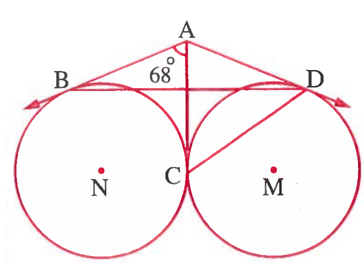
5 **[a] In the opposite figure :**

M , N are two circles touching externally at C

, $\overrightarrow{AC}$ is a common tangent of the two circles

, $\overrightarrow{AD}$ is a tangent to the circle M at D

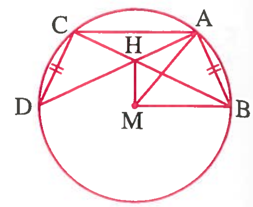
, $\overrightarrow{AB}$ is a tangent to the circle N at B , m (∠ BAC) = 68°



1 **Prove that :** AB = AD

2 **Find :** m (∠ BDC)

[b] In the opposite figure :



M is a circle where AB = CD

, m (∠ AMH) = 40°

1 **Prove that :** ABMH is a cyclic quadrilateral.

2 **Find :** m (∠ ADC)

6 El-Monofia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from those given :

1 The number of the diagonals of the pentagon ....

(a) 4 (b) 5 (c) 6 (d) 7

2 The surface area of a circle = ...

(a) π r (b) 2 π r (c) πr^2 (d) $2 \pi r^2$

3 A rhombus with diagonal lengths 6 cm. , 8 cm. has a surface area = ... cm^2.

(a) 48 (b) 28 (c) 24 (d) 12

4 If the two circles M , N are touching externally , their radii lengths are 6 cm. , 4 cm. , then MN = ... cm.

(a) 10 (b) 6 (c) 4 (d) 2

5 .... is a cyclic quadrilateral.

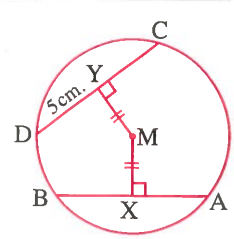
(a) Rhombus (b) Parallelogram (c) Trapezoid (d) Rectangle

6 If ABCD is a cyclic quadrilateral , m (∠ B) = 50° , then m (∠ D) = ...

(a) 130° (b) 90° (c) 50° (d) 25°

---

2 [a] In the opposite figure :

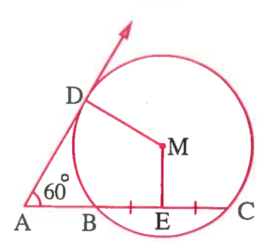


$\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$, YD = 5 cm. , MX = MY

Find with proof :

The length of $\overline{AB}$

[b] In the opposite figure :

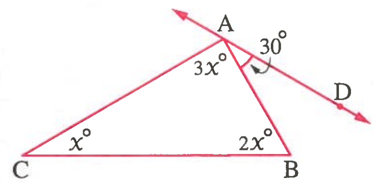


$\overrightarrow{AD}$ is a tangent to the circle M , $\overrightarrow{AB}$ intersects the circle at B and C , E is the midpoint of $\overline{BC}$, m (∠ A) = 60°

1 **Prove that :** ADME is a cyclic quadrilateral.

2 **Find :** m (∠ M)

3 [a] **In the opposite figure :**



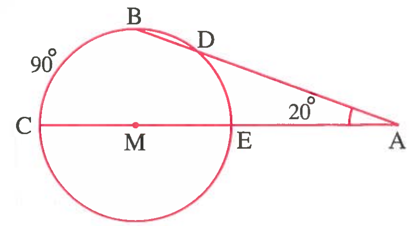
m (∠ DAB) = 30° , m (∠ BAC) = (3 X)°

, m (∠ B) = (2 X)° , m (∠ C) = (X)°

1 **Find :** the value of X

2 **Prove that :** $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , B and C

[b] **In the opposite figure :**



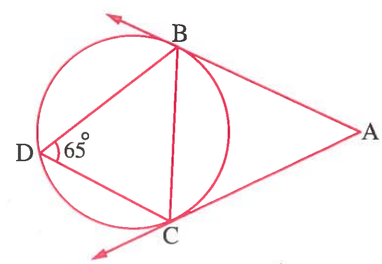
M is a circle

, m (∠ A) = 20° , m ($\overset{\frown}{CB}$) = 90°

Find with proof : m ($\overset{\frown}{DB}$)

---

4 [a] **In the opposite figure :**

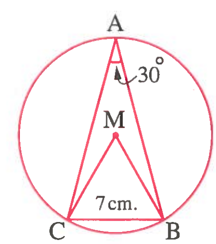


$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to

the circle at B and C

, **find with proof :** m (∠ BAC)

[b] **In the opposite figure :**

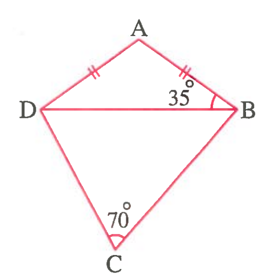


M is a circle , BC = 7 cm. , m (∠ A) = 30°

Find the circumference of the circle M $\left(\pi = \frac{22}{7}\right)$

---

5 [a] **In the opposite figure :**

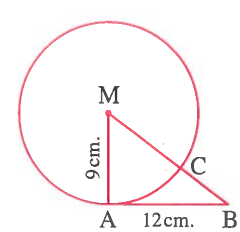


AB = AD , m (∠ ABD) = 35°

, m (∠ C) = 70°

Prove that : ABCD is a cyclic quadrilateral

[b] **In the opposite figure :**



M is a circle , $\overline{AB}$ is a tangent-segment

to the circle at A , MA = 9 cm.

, AB = 12 cm.

Find : The length of $\overline{BC}$

7 El-Gharbia Governorate

Answer the following questions :

1 Choose the correct answer from the given ones :

1 The measure of the inscribed angle drawn in a semicircle equals ....

(a) 45° (b) 90° (c) 135° (d) 180°

2 ABCD is a cyclic quadrilateral in which m (∠ A) = 2 m (∠ C)

, then m (∠ A) = ...

(a) 180° (b) 90° (c) 120° (d) 60°

3 If M is a circle of radius length r , A is a point in its plane where MA = $\frac{3}{2}$ r

, then A lies .... the circle.

(a) on (b) outside (c) inside (d) in the center of

4 The point of concurence of the medians of the triangle divides each of them in the ratio .... from the base.

(a) 1 : 3 (b) 4 : 6 (c) 2 : 1 (d) 5 : 10

5 The area of the square whose side length is 4 cm. equals .... cm^2.

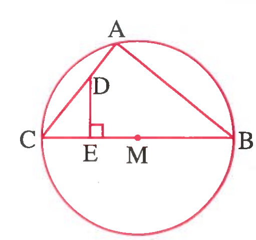
(a) 2 (b) 4 (c) 8 (d) 16

6 ABC is a right-angled triangle at A , then BC .... AC

(a) > (b) ≤ (c) = (d) twice

---

2 [a] In the opposite figure :



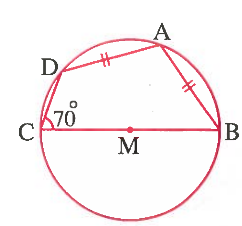
$\overline{BC}$ is a diameter in the circle M

, $\overline{DE} \perp \overline{BC}$

Prove that : 1 ABED is a cyclic quadrilateral.

2 m (∠ EDC) = $\frac{1}{2}$ m $(\overset{\frown}{AC})$

[b] In the opposite figure :

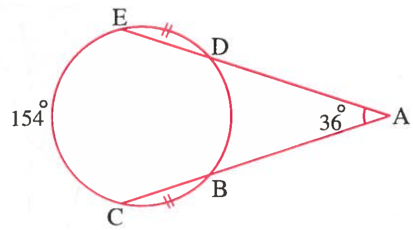


$\overline{BC}$ is a diameter in the circle M

, m (∠ C) = 70° and AB = AD

Find with proof : m (∠ ADC)

3 [a] **In the opposite figure :**

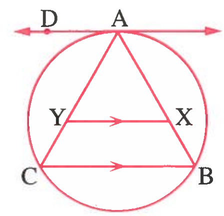


m $(\overset{\frown}{EC}) = 154°$, m $(\angle A) = 36°$

, m $(\overset{\frown}{DE})$ = m $(\overset{\frown}{BC})$

Find with proof : m $(\overset{\frown}{DE})$

[b] **In the opposite figure :**



ABC is a triangle inscribed in a circle

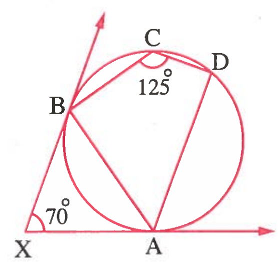
, $\overleftrightarrow{AD}$ is a tangent to the circle at A

, $X \in \overline{AB}$, $Y \in \overline{AC}$ where $\overline{XY} \parallel \overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

---

4 [a] **In the opposite figure :**

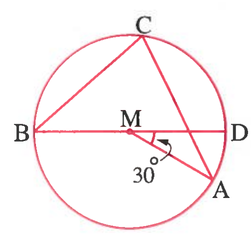


$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B , m $(\angle AXB) = 70°$ and m $(\angle DCB) = 125°$

Prove that : 1 $\overrightarrow{AB}$ bisects $\angle DAX$

2 $\overline{AD} \parallel \overrightarrow{XB}$

[b] **In the opposite figure :**



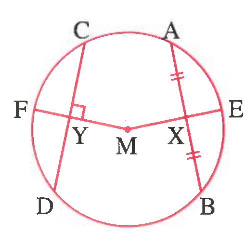
$\overline{DB}$ is a diameter in the circle M

, m $(\angle DMA) = 30°$

Find : m $(\angle ACB)$

---

5 [a] **In the opposite figure :**

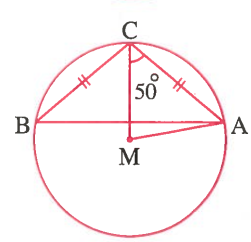


$\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

, AB = CD , X is the midpoint of $\overline{AB}$, $\overline{MY} \perp \overline{CD}$

Prove that : EX = FY

[b] **In the opposite figure :**



M is a circle , AC = BC

, m $(\angle ACM) = 50°$

Find with proof : m $(\angle MAB)$

8 El-Dakahlia Governorate

Answer the following questions : *(Calculator is permitted)*

1 [a] Choose the correct answer :

1 If the length of the longest chord in a circle is 6 cm. , then the area of the circle equals …………… cm^2.

(a) 6π (b) 9π (c) 12π (d) 36π

2 A circle its centre is the origin point and its radius length is 5 length units. Which of the following points does not belong to the circle ?

(a) (5 , 5) (b) (0 , 5) (c) (5 , 0) (d) (0 , – 5)

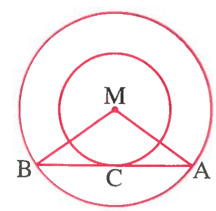
3 The ratio between the measure of the inscribed angle : the measure of the central angle that has the same subtended arc is ……………

(a) 2 : 1 (b) 1 : 3 (c) 1 : 1 (d) 2 : 4

[b] In the opposite figure :

Two circles are concentric at M , their radii lengths are 5 cm. , 3 cm. , $\overline{AB}$ is a chord in the greater circle and touches the smaller circle at C

Find : The perimeter of Δ MAB



2 [a] Choose the correct answer :

1 A circle with radius length r , the straight line L is at a distance x from its center where $x \in]0 , r[$, then the straight line L is ……………

(a) a secant to the circle. (b) a tangent to the circle.

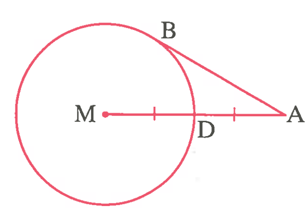
(c) outside the circle. (d) passing through the centre.

2 If ABCD is a cyclic quadrilateral , then m ($\angle$ A) + m ($\angle$ C) – 100° = ……………

(a) 180° (b) 100° (c) 90° (d) 80°

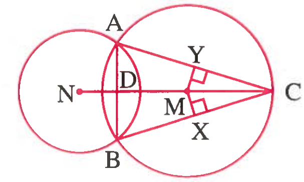
3 **In the opposite figure :**

A circle of radius length r , if $\overline{AB}$ is a tangent-segment to the circle at B , $\overline{AM}$ intersects the circles at D where AD = DM , then AB = ……………



(a) 2 r (b) $\frac{\sqrt{3}}{2}$ r (c) $\sqrt{3}$ r (d) r

[b] In the opposite figure :



The circle M $\cap$ The circle N = $\{A , B\}$

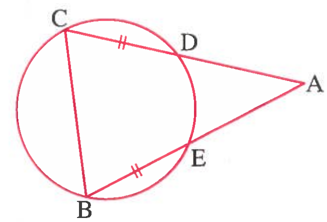
, $\overleftrightarrow{MN} \cap \overline{AB} = \{D\}$, $C \in \overleftrightarrow{MN}$

, if $\overline{MX} \perp \overline{BC}$, $\overline{MY} \perp \overline{AC}$

Prove that : MX = MY

---

3 **[a] In the opposite figure :**

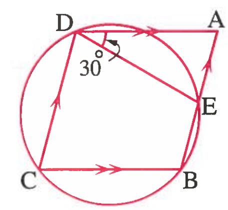


$\overline{CD}$, $\overline{BE}$ are two chords equal in length

, $\overrightarrow{CD} \cap \overrightarrow{BE} = \{A\}$

Prove that : AD = AE

[b] In the opposite figure :

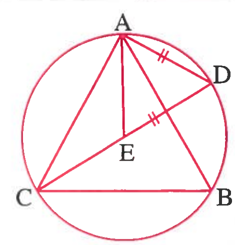


ABCD is a parallelogram , the circle is passing through the points B , C , D cuts $\overline{AB}$ at E , m (∠ ADE) = 30°

Find : m (∠ B)

---

4 **[a] In the opposite figure :**

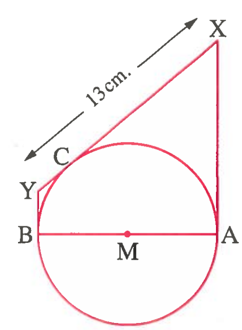


ABC is an equilateral triangle inscribed in a circle , $E \in \overline{DC}$, where AD = DE

Prove that : 1 The triangle ADE is equilateral.

2 m (∠ DCB) = m (∠ EAC)

[b] In the opposite figure :



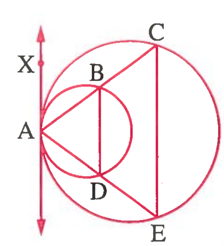
$\overline{AB}$ is a diameter in a circle M whose radius length is 5 cm.

, if C ∈ the circle M , a tangent to the circle is drawn at point C and cut the two tangents of the circle at A , B in X , Y where XY = 13 cm.

Find : The area of the figure AXYB

---

5 **[a] In the opposite figure :**



Two circles are touching internally at A

, $\overleftrightarrow{AX}$ is common tangent to them at A

, $\overleftrightarrow{AB}$ and $\overleftrightarrow{AD}$ intersect the smaller circle at B , D and intersect the greater circle at C , E

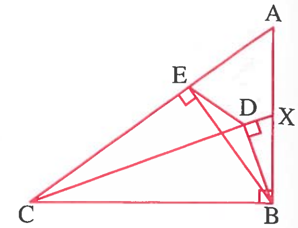
Prove that : $\overline{DB}$ // $\overline{EC}$

[b] In the opposite figure :

Δ ABC is a right angled-triangle at B

, $\overline{BE} \perp \overline{AC}$, $\overline{BD} \perp \overline{XC}$

Prove that : AXDE is a cyclic quadrilateral.



9 Suez Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from those given :

1 The parallelogram whose diagonals are equal in length and not perpendicular is ....

(a) a rhombus. (b) a rectangle. (c) a trapezium. (d) a square.

2 M and N are two circles touching externally their radii lengths are 5 cm. , and 3 cm. respectively , then MN = ... cm.

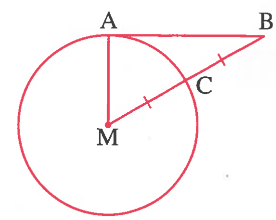
(a) 2 (b) 3 (c) 5 (d) 8

3 **In the opposite figure :**

$\overline{AB}$ is a tangent-segment to the circle M

, $\overline{MB}$ intersects the circle at C , C is the midpoint of $\overline{MB}$

, then m $(\overset{\frown}{AC})$ = ...



(a) 30° (b) 45° (c) 60° (d) 90°

4 The circumference of the circle whose diameter length 6 cm. is .... cm.

(a) 3π (b) 6π (c) 2π (d) $\frac{3}{2}\pi$

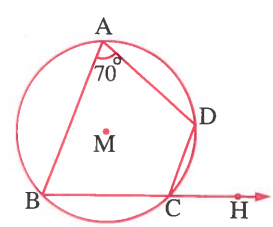
5 The inscribed angle drawn in semicircle is ....

(a) acute. (b) right. (c) obtuse. (d) straight.

6 **In the opposite figure :**

m (∠ A) = 70°

, then m (∠ HCD) = ...



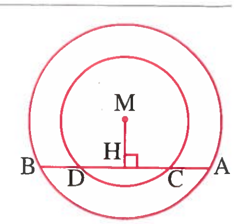
(a) 70° (b) 140°

(c) 35° (d) 110°

2 [a] In the opposite figure :

$\overline{AB}$ is a chord in the greater circle cuts the smaller circle at C and D , $\overline{MH} \perp \overline{AB}$

Prove that : AC = BD

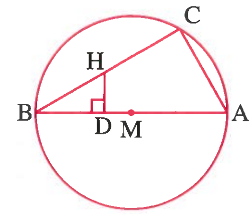


[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, $\overline{HD} \perp \overline{AB}$

Prove that : The figure ACHD is cyclic quadrilateral.



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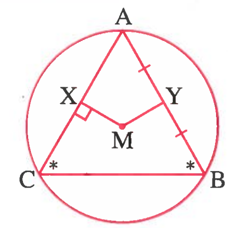
3 **[a] In the opposite figure :**

ABC is an inscribed triangle in the circle M , m (∠ B) = m (∠ C)

, Y is the midpoint of $\overline{AB}$, $\overline{MX} \perp \overline{AC}$

Prove that : MX = MY

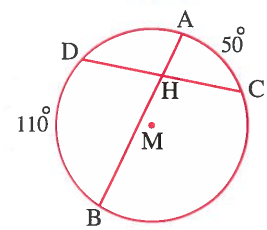


[b] In the opposite figure :

$\overline{AB} \cap \overline{CD} = \{H\}$, m $(\overset{\frown}{AC}) = 50°$

, m $(\overset{\frown}{BD}) = 110°$

Find : m (∠ AHC)



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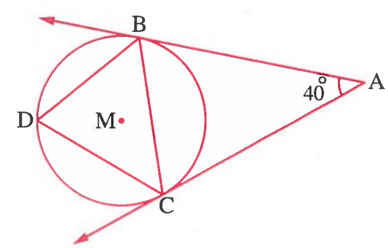
4 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments

to the circle M at B and C , m (∠ A) = 40°

, find with proof : m (∠ D)

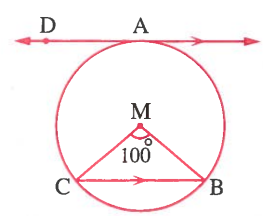


[b] In the opposite figure :

$\overleftrightarrow{AD}$ is a tangent to the circle M at A

, $\overleftrightarrow{AD}$ // $\overline{BC}$, m (∠ M) = 100°

Find : m $(\overset{\frown}{AB})$



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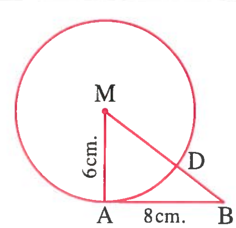
5 **[a] In the opposite figure :**

$\overline{AB}$ is a tangent-segment to the circle M at A

, $\overline{MB}$ cuts the circle M at D , AM = 6 cm. , AB = 8 cm.

Find : BD



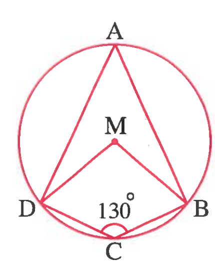
[b] In the opposite figure :

The circle M passes through the vertices

of the quadrilateral ABCD , m (∠ C) = 130°

Find : 1 m (∠ A)

2 m (∠ BMD)



10 Damietta Governorate

Answer the following questions : (Calculator is permitted)

1 Choose the correct answer from those given :

1 If two straight lines intersect , then each two vertically opposite angles are ....

(a) equal in measure. (b) complementary.

(c) supplementary. (d) corresponding.

2 The square whose area is 100 cm^2 its diagonal length equal .... cm.

(a) $2\sqrt{10}$ (b) 10 (c) $10\sqrt{2}$ (d) 50

3 If ABCD is a cyclic quadrilateral , then m ($\angle$ A) + m ($\angle$ C) – 50° = ...

(a) 50° (b) 130° (c) 150° (d) 180°

4 The sum of lengths of any two sides in a triangle is .... the length of the third side.

(a) twice (b) equal (c) less than (d) greater than

5 If the two circles M , N are touching externally , the radius length of one of them is 3 cm. , and MN = 8 cm. , then the radius length of the other circle equals .... cm.

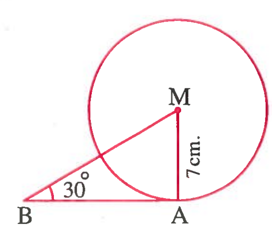
(a) 5 (b) 6 (c) 11 (d) 12

6 The inscribed angle drawn in a semicircle is .... angle.

(a) a straight (b) an obtuse (c) a right (d) an acute

---

2 [a] In the opposite figure :

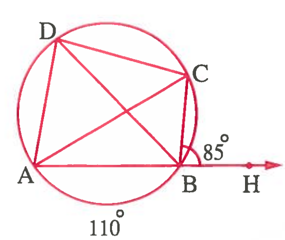


$\overline{AB}$ is a tangent-segment to the circle M at A

, m ($\angle$ B) = 30° and AM = 7 cm.

Find with proof : the length of $\overline{MB}$

[b] In the opposite figure :



If H $\in \overrightarrow{AB}$, H $\notin \overline{AB}$

, m ($\angle$ HBC) = 85° , m $(\overset{\frown}{AB})$ = 110°

Find with proof :

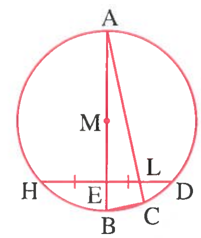
1 m ($\angle$ ADB) 2 m ($\angle$ BAC)

3 [a] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, E is the midpoint of $\overline{HD}$

Prove that : LCBE is a cyclic quadrilateral



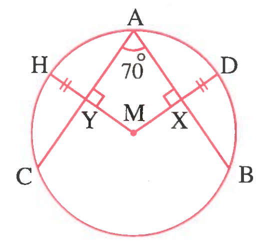
[b] **In the opposite figure :**

A circle M , $\overline{MD} \perp \overline{AB}$, $\overline{MH} \perp \overline{AC}$

, XD = YH , m (∠ BAC) = 70°

1 **Find :** m (∠ XMY)

2 **Prove that :** AB = AC



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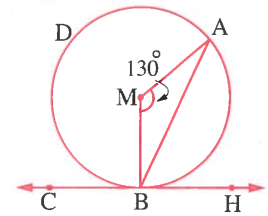
4 [a] **In the opposite figure :**

$\overleftrightarrow{HC}$ is a tangent to the circle M at B

, D ∈ the major arc $\overset{\frown}{AB}$, m (∠ AMB) = 130°

Find with proof : 1 m (∠ ABH)

2 m ($\overset{\frown}{ADB}$)

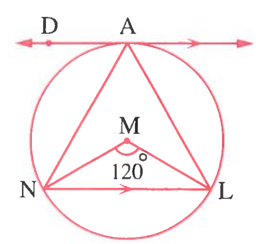


[b] **In the opposite figure :**

$\overleftrightarrow{AD}$ is a tangent to the circle M at A

, $\overleftrightarrow{AD}$ // $\overline{LN}$, m (∠ LMN) = 120°

Prove that : The triangle ALN is an equilateral triangle.



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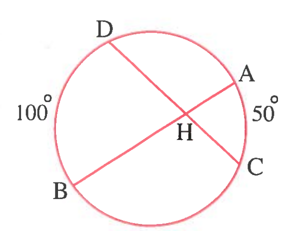
5 [a] **In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{H\}$, m ($\overset{\frown}{AC}$) = 50°

, m ($\overset{\frown}{DB}$) = 100°

Find with proof : m (∠ AHC)



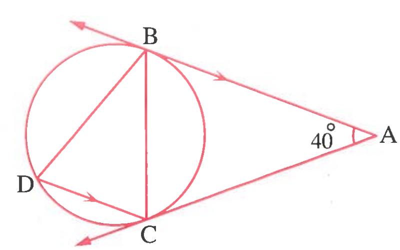
[b] **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle

at B and C , $\overrightarrow{AB}$ // $\overline{CD}$, m (∠ A) = 40°

1 **Find with proof :** m (∠ BDC)

2 **Prove that :** $\overline{BD}$ is a tangent-segment to the circumcircle of the triangle ACB



11 Kafr El-Sheikh Governorate

Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer from those given :

1 The measure of the exterior angle of the equilateral triangle equals ....

(a) 60° (b) 120° (c) 90° (d) 150°

2 An angle of measure 30° is complementary to the angle whose measure is ....

(a) 30° (b) 40° (c) 150° (d) 60°

3 ABCD is a cyclic quadrilateral , m (∠ A) = 80° , then m (∠ C) = ...

(a) 80° (b) 100° (c) 10° (d) 90°

4 The inscribed angle drawn in a semicircle is .... angle.

(a) an acute (b) a right (c) an obtuse (d) a straight

5 ABCD is a parallelogram , m (∠ A) + m (∠ C) = 260° , then m (∠ B) = ...

(a) 100° (b) 50° (c) 130° (d) 60°

6 M and N are two intersecting circles , the lengths of their diameters are 10 cm. , 4 cm.

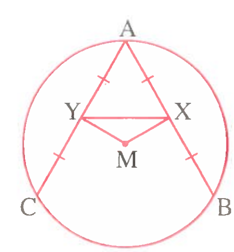
, then MN ∈ ....

(a)]6 , 14[(b)]3 , 7[(c) [6 , 14] (d) [3 , 7]

2 [a] In the opposite figure :

$\overline{AB}$ and $\overline{AC}$ are two equal chords in length in the circle M , X and Y are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively

Prove that : m (∠ YXB) = m (∠ XYC)

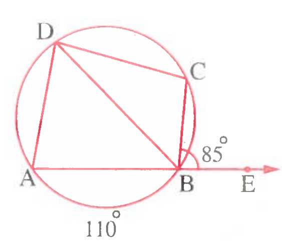


[b] In the opposite figure :

$E \in \overrightarrow{AB}$, $E \notin \overline{AB}$, m $(\overset{\frown}{AB})$ = 110°

, m (∠ CBE) = 85°

Find with proof : m (∠ BDC)



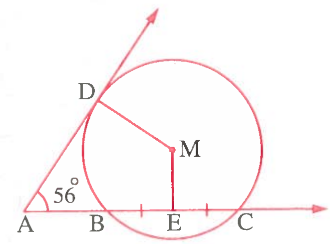
3 [a] **In the opposite figure :**

$\overrightarrow{AD}$ is a tangent to the circle M

, $\overrightarrow{AC}$ intersects the circle M at B and C

, E is the midpoint of $\overline{BC}$, m (∠ A) = 56°

, **find with proof :** m (∠ DME)



[b] By using the geometric tools draw the triangle ABC in which AB = 4 cm. , BC = 5 cm. and CA = 6 cm. , then draw circle passing through the points A , B and C , what is the kind of triangle ABC with respect to the measure of its angles ?

---

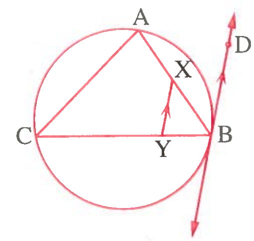
4 [a] **In the opposite figure :**

ABC is a triangle inscribed in a circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, X ∈ $\overline{AB}$, Y ∈ $\overline{BC}$, where $\overline{XY}$ // $\overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.



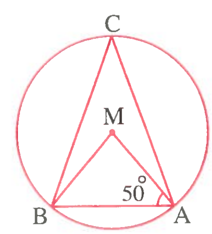
[b] **In the opposite figure :**

The circle M where m (∠ MAB) = 50°

Find with proof :

1 m (reflex ∠ AMB)

2 m (∠ C)



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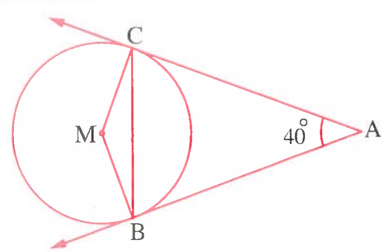
5 [a] **In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents of the circle M at B and C respectively

, where m (∠ A) = 40°

Find with proof : 1 m (∠ ABC) 2 m (∠ MCB)



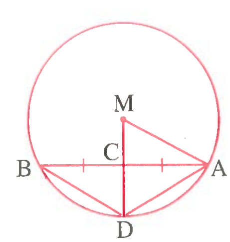
[b] **In the opposite figure :**

The circle M with radius length 13 cm.

, $\overline{AB}$ is a chord of length 24 cm.

, C is a midpoint of $\overline{AB}$, $\overrightarrow{MC}$ ∩ the circle M = {D}

Find : The area of the triangle ADB



12 El-Fayoum Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from those given :

1 The measure of the inscribed angle drawn in a semicircle = ...

(a) 30° (b) 60° (c) 90° (d) 120°

2 If the area of a square is 25 cm^2, then its perimeter = ... cm.

(a) 5 (b) 10 (c) 20 (d) 25

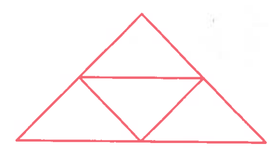
3 If 2 cm. , 4 cm. , X cm. are the lengths of the sides of a triangle , then X can be equal to .... cm.

(a) 1 (b) 2 (c) 4 (d) 6

4 If ABCD is a cyclic quadrilateral , m (∠ ABD) = 30° , then m (∠ ACD) ....

(a) 30° (b) 60° (c) 90° (d) 150°

5 The number of the triangle in the opposite figure = ...



(a) 2 (b) 3

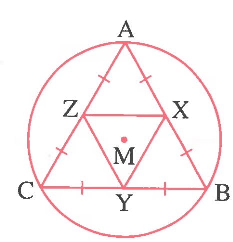
(c) 4 (d) 5

6 If the radii lengths of the two circle M , N are 5 cm. , 3 cm. and MN = 4 cm. , then the two circle M , N are ....

(a) distant. (b) touching internally.

(c) touching externally. (d) intersecting.

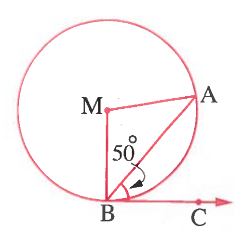
2 [a] In the opposite figure :



M is the circumcircle of the equilateral triangle ABC , X , Y , Z are the midpoints of $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$ respectively.

Prove that : M is the centre of the circumcircle of the triangle XYZ

[b] In the opposite figure :



$\overrightarrow{BC}$ is a tangent to the circle M

, m (∠ ABC) = 50°

Find : m (∠ AMB)

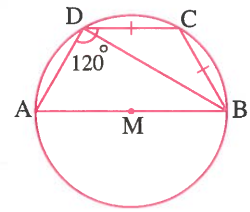
3 [a] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, CD = CB , m (∠ ADC) = 120°

Find : 1 m (∠ DBC)

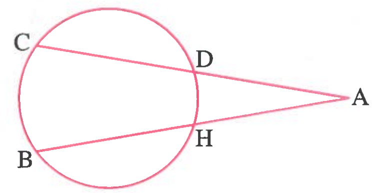
2 m (∠ BAD)



[b] **In the opposite figure :**

m $(\overset{\frown}{BC})$ = 80° , m $(\overset{\frown}{HD})$ = 30°

Find : m (∠ A)



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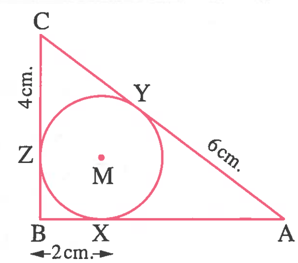
4 [a] **In the opposite figure :**

$\overline{AB}$, $\overline{BC}$, $\overline{AC}$ touch the circle

at X , Z , Y respectively , BX = 2 cm.

, CZ = 4 cm. , AY = 6 cm.

Find : the area and the perimeter of triangle ABC



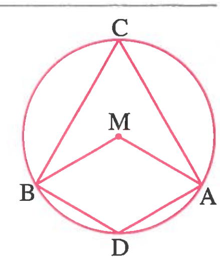
[b] If $\overline{AB}$ is a line segment , AB = 5 cm. can we draw a cricle of a radius length = 6 cm. and passing through the two points A , B ? How many circles we can draw ?

---

5 [a] **In the opposite figure :**

m $(\overset{\frown}{AC})$ = m $(\overset{\frown}{CB})$ = m $(\overset{\frown}{BA})$

, **find :** m (∠ ADB)



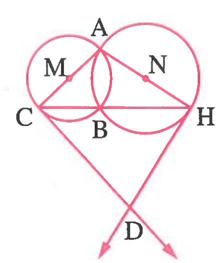
[b] **In the opposite figure :**

M , N are two circles intersting at A , B

, $\overrightarrow{HD}$ is a tangent to the circle N at H

, $\overrightarrow{CD}$ is a tangent to the circle M at C

Prove that : AHDC is a cyclic quadrilateral.



13 Souhag Governorate

Answer the following questions : *(Calculator is allowed)*

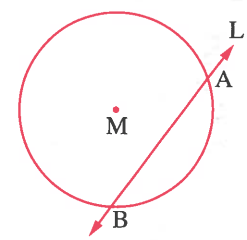
1 **Choose the correct answer from those given :**

1 In the rhombus the lengths of the two diagonals are 6 cm. , 8 cm.

, then its area = ... cm^2.

(a) 48 (b) 24 (c) 14 (d) 28

2 **In the opposite figure :**



If the straight line L $\cap$ the surface of the circle M = ...

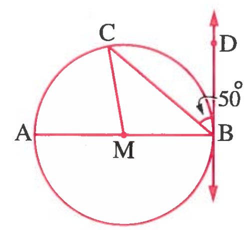
(a) $\{A , B\}$ (b) $\varnothing$

(c) $\overline{AB}$ (d) $\overleftrightarrow{AB}$

3 If C $\in$ axis of symmetry of $\overline{AB}$, then $\overline{CA}$ .... $\overline{CB}$

(a) = (b) // (c) $\perp$ (d) $\equiv$

4 **In the opposite figure :**



$\overleftrightarrow{BD}$ is tangent of the circle M at B , $\overline{AB}$ is diameter in the circle , if m ($\angle$ CBD) = 50°

, then m ($\angle$ AMC) = ...

(a) 25° (b) 50° (c) 100° (d) 80°

5 In Δ ABC , if $(AC)^2 > (AB)^2 + (BC)^2$, then $\angle$ C is ....

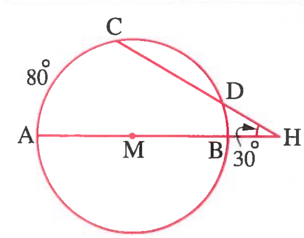
(a) acute. (b) right. (c) obtue. (d) straight.

6 If ABCD is a cyclic quadrilateral , then m ($\angle$ A) + m ($\angle$ C) – 80° = ...

(a) 60° (b) 80° (c) 100° (d) 180°

---

2 [a] **In the opposite figure :**



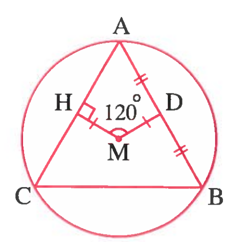
$\overline{AB}$ is a diamter the circle M

, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{H\}$

, m ($\angle$ AHC) = 30° , m $(\overset{\frown}{AC})$ = 80°

Find : m $(\overset{\frown}{CD})$

[b] **In the opposite figure :**



D is midpoint of $\overline{AB}$, $\overline{MH} \perp \overline{AC}$

, MD = MH , m ($\angle$ DMH) = 120°

Prove that : Δ ABC is an equilateral triangle.

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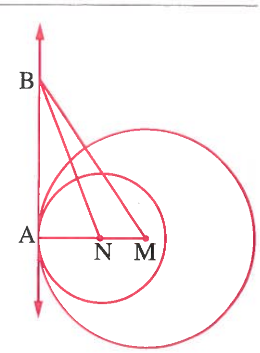
3 [a] **In the opposite figure :**



M and N are two circles the lengths of their radii are 10 cm. and 6 cm. respectively and they are touching internally at A , $\overleftrightarrow{AB}$ is common tangent for both at A

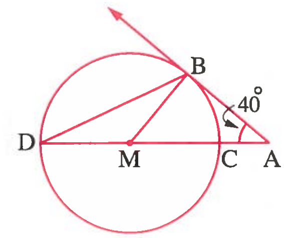
, the area of Δ BMN = 24 cm^2.

Find : The length of $\overline{AB}$

[b] In the opposite figure :

A is a point outside the circle M , $\overrightarrow{AB}$ is a tangent to the circle at B , m (∠ A) = 40° , $\overrightarrow{AM}$ intersects the circle M at C and D respectively.

Find with proof : m (∠ BDC)



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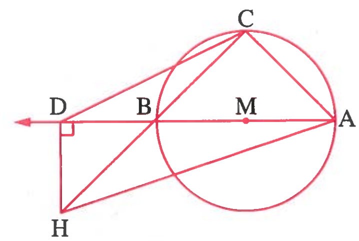
4 **[a] In the opposite figure :**

$\overline{AB}$ is diameter in circle M , D ∈ $\overrightarrow{AB}$, D ∉ $\overline{AB}$, $\overrightarrow{DH} \perp \overrightarrow{AB}$, C ∈ $\overset{\frown}{AB}$, $\overrightarrow{CB} \cap \overrightarrow{DH}$ = {H}

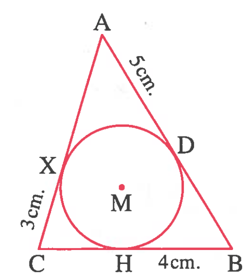
Prove that : ACDH is a cyclic quadrilateral.



[b] In the opposite figure :

M is the inscribed circle of the triangle ABC , AD = 5 cm. , BH = 4 cm. , CX = 3 cm.

Find : The perimeter of Δ ABC



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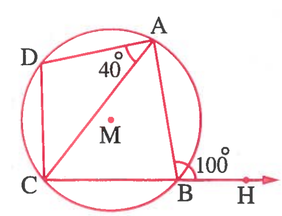
5 **[a] In the opposite figure :**

ABCD is a cyclic quadrilateral inscribed in the circle M , m (∠ ABH) = 100° , m (∠ DAC) = 40°

Prove that : m ($\overset{\frown}{AD}$) = m ($\overset{\frown}{CD}$)

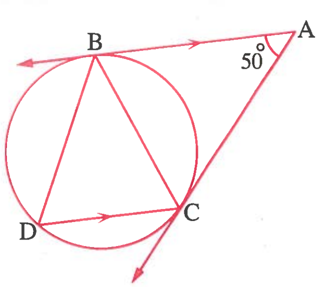


[b] In the opposite figure :

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C , m (∠ A) = 50° , $\overrightarrow{AB}$ // $\overline{CD}$

1 **Find :** m (∠ BDC)

2 **Prove that :** $\overline{BD}$ is tangent-segment to the circle passing through the vertices of Δ ABC



14 Qena Governorate

Answer the following questions : *(Calculator is perimetted)*

1 Choose the correct answer from those given :

1. It is possible to drawing a circle passing through the vertices of ....
   (a) a rhombus. (b) a rectangle. (c) a trapezium. (d) a parallelogram.

2. The number of axes of a semicircle is ....
   (a) an infinit number. (b) 2
   (c) 1 (d) zero

3. The measure of the inscribed angle drawing in a semicircle equals ....
   (a) 45° (b) 90° (c) 135° (d) 180°

4. The length of the side opposite to the angle of measure 30° in the right-angled triangle = ... the length of the hypotenuse.
   (a) half (b) fourth (c) third (d) twice

5. If A , B are two points in the plane and AB = 8 cm. , then the length of diameter of the smallest circle passing through the two points A and B is .... cm.
   (a) 2 (b) 4 (c) 16 (d) 8

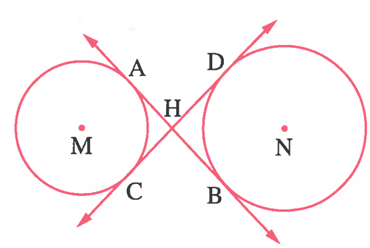
6. The lengths of two diagonals of a rhombus are 8 cm. and 6 cm.
   , then its perimeter = ... cm.
   (a) 14 (b) 28 (c) 40 (d) 20

---

2 [a] **In the opposite figure :**

M , N are two distant circles , $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are drawn to touch the circle M at A , C and the circle N at B , D

Prove that : AB = CD



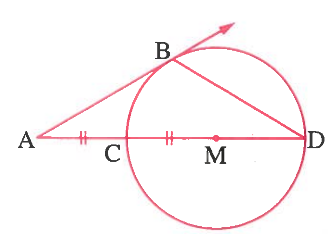
[b] **In the opposite figure :**

$\overline{CD}$ is a diameter in the circle M

, A $\in \overrightarrow{DC}$ where AC = CM

, $\overrightarrow{AB}$ is a tangent to the circle M at B

Find : m (∠ ABD)



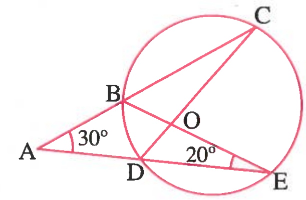
3 **[a] In the opposite figure :**

$\overline{BC}$, $\overline{DE}$ are two chords in the circle

, $\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$, where m $(\angle A) = 30°$

, m $(\angle AEB) = 20°$

Find : m $(\angle CDE)$, m $(\angle COE)$

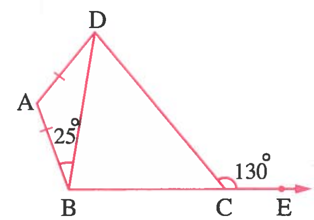


[b] In the opposite figure :

$AB = AD$, m $(\angle ABD) = 25°$

, $E \in \overrightarrow{BC}$, m $(\angle DCE) = 130°$

Prove that : The figure ABCD is a cyclic quadrilateral.



---

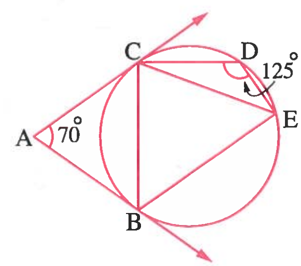
4 **[a] In the opposite figure :**

BCDE is a cyclic quadrilateral in which

m $(\angle CDE) = 125°$, $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C , m $(\angle A) = 70°$

Prove that : 1 Δ BCE is an isosceles triangle

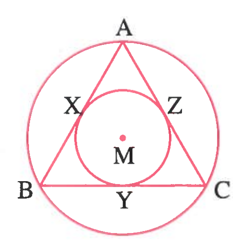
2 $\overrightarrow{AC} \,//\, \overline{BE}$



[b] In the opposite figure :

Two concentric circles at M , the greatest passing through the vertices of Δ ABC and the smallest touches its sides $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ at X , Y , Z respectively

Prove that : Δ ABC is an equilateral triangle



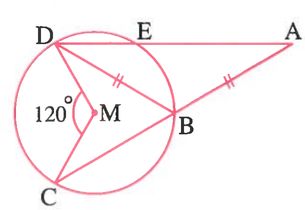
---

5 **[a] In the opposite figure :**

$AB = BD$, m $(\angle CMD) = 120°$

, $\overrightarrow{CB} \cap \overrightarrow{DE} = \{A\}$

Find : m $(\angle A)$



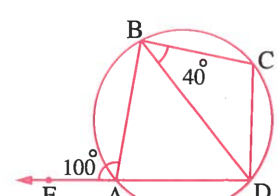
[b] In the opposite figure :

ABCD is a cyclic quadrilateral

, $E \in \overrightarrow{DA}$ where m $(\angle BAE) = 100°$

, m $(\angle CBD) = 40°$

Prove that : m $(\overset{\frown}{BC})$ = m $(\overset{\frown}{CD})$



15 Aswan Governorate

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1. ABC is a triangle where m (∠ A) = m (∠ C) – m (∠ B) , then m (∠ C) ……………

 (a) 45° (b) 60° (c) 90° (d) 180°

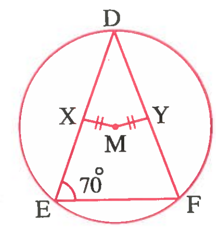
2. **In the opposite figure :**

 M is a circle , where MX = MY

 , m (∠ E) = 70° , then m (∠ D) = ……………

 (a) 70° (b) 35°

 (c) 40° (d) 55°



3. The two straight lines L_1 , L_2 , if $L_1 \cap L_2 = \varnothing$, then the two straight lines are ……………

 (a) intersecting. (b) parallel. (c) congruent. (d) equal.

4. If 9 cm. , 4 cm. are the lengths of two radii of the two intersecting circles , then the distance between their centres ∈ ……………

 (a)]5 , 13[(b) [5 , 13] (c)]4 , 9[(d) [4 , 9]

5. ABCD is a parallelogram in which , m (∠ A) = 100° , then m (∠ B) + m (∠ D) = ……………

 (a) 240° (b) 80° (c) 180° (d) 160°

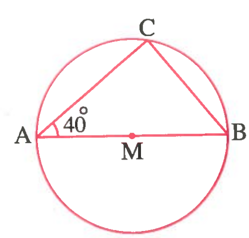
6. **In the opposite figure :**

 $\overline{AB}$ is a diameter in the circle M

 , m (∠ A) = 40° , then m (∠ B) = ……………

 (a) 40° (b) 50°

 (c) 70° (d) 20°



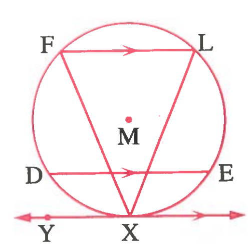
2

[a] M is a circle whose radius length 5 cm. where A and B are two points belong to the circle where m (∠ AMB) = 108° , find the length of $\overset{\frown}{AB}$ (Where π = 3.14)

[b] **In the opposite figure :**

$\overleftrightarrow{XY}$ is a tangent to the circle M at X , $\overline{DE}$ and $\overline{FL}$ are two chords in the circle where $\overline{FL}$ // $\overline{DE}$ // $\overleftrightarrow{XY}$

Prove that : XF = XL

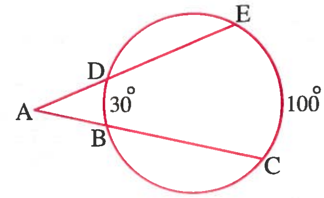


3 [a] **In the opposite figure :**

$\overrightarrow{ED} \cap \overrightarrow{CB} = \{A\}$

where m $(\overset{\frown}{BD}) = 30°$, m $(\overset{\frown}{EC}) = 100°$

Find : m (∠ A)

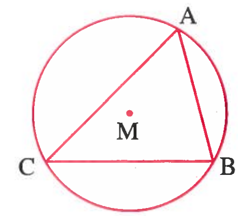


[b] **In the opposite figure :**

ABC is an inscribed triangle in the circle M

where m $(\overset{\frown}{AB})$: m $(\overset{\frown}{BC})$: m $(\overset{\frown}{AC}) = 3 : 4 : 5$

Find : m (∠ ABC)



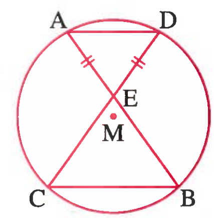
---

4 [a] **In the opposite figure :**

M is a circle , $\overline{AB} \cap \overline{CD} = \{E\}$

, EA = ED

Prove that : EB = EC

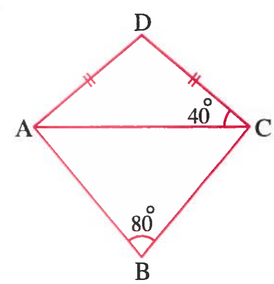


[b] **In the opposite figure :**

AD = DC , m (∠ ACD) = 40°

, m (∠ ABC) = 80°

Prove that : ABCD is a cyclic quadrilateral



---

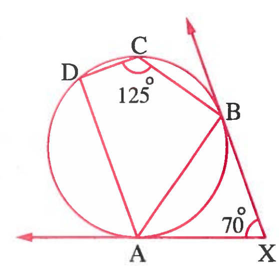
5 [a] **In the opposite figure :**

$\overrightarrow{XA}$, $\overrightarrow{XB}$ are two tangents to the circle

at A and B , m (∠ AXB) = 70°

, m (∠ DCB) = 125°

Prove that : $\overline{AD} \mathbin{//} \overrightarrow{XB}$



[b] **In the opposite figure :**

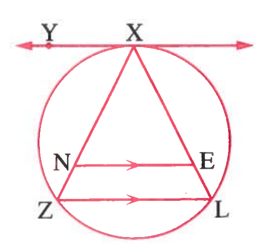
XZL is a triangle inscribed in a circle

, $\overleftrightarrow{XY}$ is a tangent to the circle at X

, N ∈ $\overline{XZ}$, E ∈ $\overline{XL}$ where $\overline{NE} \mathbin{//} \overline{ZL}$

Prove that : $\overleftrightarrow{XY}$ is a tangent to the circle

passing through the points X , N and E



Exam 1 Port Said 2024

First Multiple choice questions

Choose the correct answer from those given :

1 The tangent to a circle , of diameter length equals 6 cm. , is at distance .... cm. from its centre.

(a) 12 (b) 6 (c) 3 (d) 2

---

2 In the cyclic quadrilateral each two opposite angles are ....

(a) supplementary. (b) complementary. (c) equal in measure. (d) alternate.

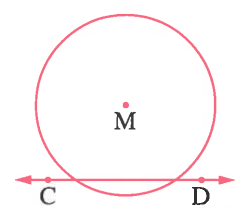
---

3 The complementary angle to the angle whose measure 70° equals ....

(a) 20° (b) 70° (c) 110° (d) 180°

---

4 **In the opposite figure :**



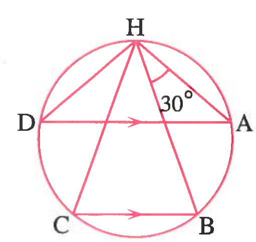
$\overleftrightarrow{CD} \cap$ the surface of the circle M = ...

(a) $\varnothing$ (b) $\{C , D\}$

(c) $\overline{CD}$ (d) $\overleftrightarrow{CD}$

---

5 **In the opposite figure :**



$\overline{AD} \mathbin{//} \overline{BC}$, m (∠ AHB) = 30°

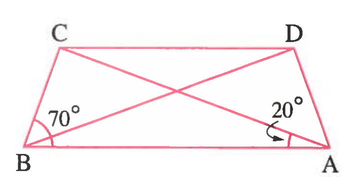
, then m (∠ CHD) = ...

(a) 10° (b) 30°

(c) 50° (d) 80°

---

6 **In the opposite figure :**



ABCD is cyclic quadrilateral if m (∠ BAC) = 20°

, m (∠ ABC) = 70° , then m (∠ ADB) = ...

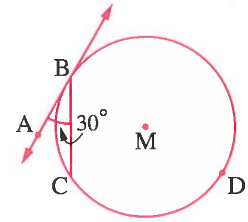
(a) 20° (b) 70°

(c) 80° (d) 90°

7 **In the opposite figure :**

$\overleftrightarrow{AB}$ is a tangent for the circle M , m (∠ CBA) = 30°

, then m $(\overset{\frown}{BDC})$ = ...



(a) 30° (b) 60°

(c) 240° (d) 300°

---

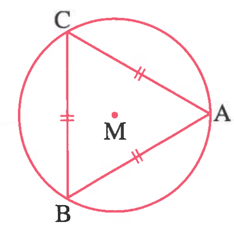
8 The length of the radius of the smallest circle that passing through the two endpoints of a line segment .... half its length.

(a) equals (b) more than (c) less than (d) twice

---

9 **In the opposite figure :**

Δ ABC is an equilateral triangle , if the length of $\overset{\frown}{AB}$ = 8 cm. , then the circumference of the circumcircle of the triangle ABC = ... cm.



(a) 48 (b) 24

(c) 16 (d) 8

---

10 The area of the square whose side length = 5 cm. is .... cm^2

(a) 10 (b) 15 (c) 20 (d) 25

---

11 The measure of the arc which is half the measure of the circle = ...

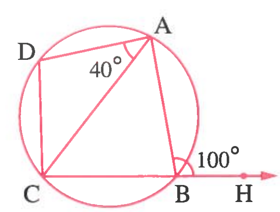
(a) 45° (b) 90° (c) 180° (d) 360°

---

12 **In the opposite figure :**

If H ∈ $\overrightarrow{CB}$, m (∠ ABH) = 100° , m (∠ CAD) = 40°

, then m $(\overset{\frown}{AD})$ = m (..............)



(a) $\overset{\frown}{CD}$ (b) $\overset{\frown}{DCB}$

(c) $\overset{\frown}{BC}$ (d) $\overset{\frown}{BAD}$

---

13 Measure of the inscribed angle which is drawn in semicircle equals ....

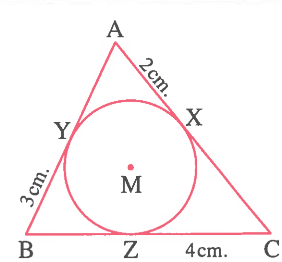
(a) 60° (b) 90° (c) 120° (d) 180°

---

14 **In the opposite figure :**

If AX = 2 cm. , BY = 3 cm. , ZC = 4 cm.

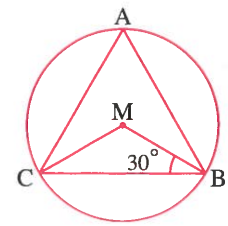
, then the perimeter of the triangle ABC = ... cm.



(a) 9 (b) 12

(c) 18 (d) 24

15 **In the opposite figure :**



Δ ABC is inscribed on the circle M , m ($\angle$ MBC) = 30°

, then m ($\angle$ A) = ...

(a) 30° (b) 60°

(c) 90° (d) 120°

---

16 A circle can be drawn passes by the vertices of ....

(a) rhombus. (b) rectangle. (c) trapezium. (d) parallelogram.

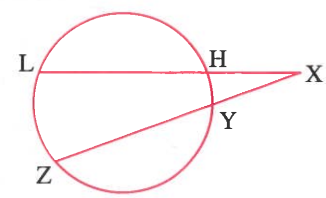
---

17 In Δ ABC , m ($\angle$ A) = 50° , m ($\angle$ B) = 70° , then the number of its axes of symmetry is ....

(a) zero (b) one (c) two (d) three

---

18 **In the opposite figure :**



If m $(\overset{\frown}{LZ})$ = 70° , m $(\overset{\frown}{HY})$ = 30°

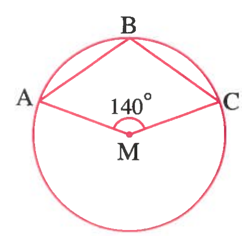
, then m ($\angle$ X) = ...

(a) 100° (b) 50°

(c) 40° (d) 20°

---

19 **In the opposite figure :**



M is a circle , m ($\angle$ AMC) = 140°

, then m ($\angle$ ABC) = ...

(a) 360° (b) 220°

(c) 110° (d) 40°

---

20 Number of common tangents for two distant circles is ....

(a) 1 (b) 2 (c) 3 (d) 4

---

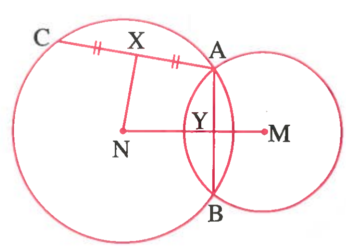
21 Two intersecting circles M and N , their radii lengths are 3 cm. and 4 cm.

, then MN $\in$ ....

(a)]7 , ∞[(b)]1 , ∞[(c)]0 , 1[(d)]1 , 7[

Second Essay questions

22 **In the opposite figure :**



AB = AC , X is the midpoint of $\overline{AC}$

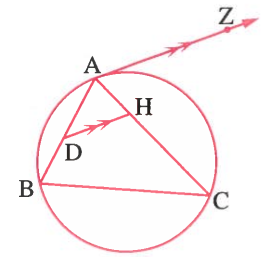
, $\overline{AB} \cap \overline{MN} = \{Y\}$

Show that : NY = NX

23 **In the opposite figure :**

$\overrightarrow{AZ} \parallel \overline{DH}$, $\overrightarrow{AZ}$ is a tangent to the circle

Show that : BCHD is a cyclic quadrilateral.



---

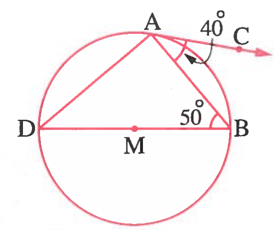
24 **In the opposite figure :**

$\overline{BD}$ is a diameter of the circle M

, m (∠ BAC) = 40° , m (∠ B) = 50°

Show that :

$\overrightarrow{AC}$ is a tangent to the circle M at A



Exam 2 Port Said 2023

First Multiple choice questions

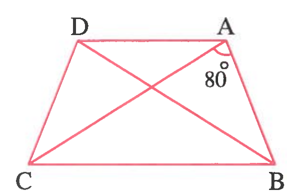
Choose the correct answer from those given :

1 **In the opposite figure :**

ABCD is a cyclic quadrilateral

in which m (∠ BAC) = 80°

, then m (∠ BDC) = ...



(a) 40° (b) 80° (c) 90° (d) 100°

---

2 The chord which passes through the centre of the circle is called ....

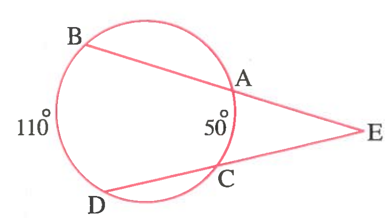
(a) tangent. (b) secant. (c) diameter. (d) radius.

---

3 **In the opposite figure :**

m $(\overset{\frown}{AC})$ = 50° , m $(\overset{\frown}{BD})$ = 110°

, then m (∠ E) = ...



(a) 10° (b) 20°

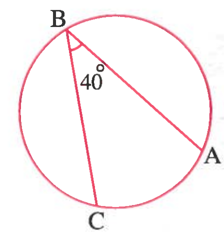
(c) 30° (d) 40°

---

4 If the point A ∈ the circle M which its diameter of length 6 cm. , then MA = ... cm.

(a) 3 (b) 4 (c) 5 (d) 6

5 **In the opposite figure :**



m (∠ ABC) = 40°

, then m $(\overset{\frown}{AC})$ = ...

(a) 20° (b) 40°

(c) 60° (d) 80°

---

6 The supplementary for the angle whose measure 70° equals ....

(a) 20° (b) 70° (c) 110° (d) 180°

---

7 The two tangents drawn from the endpoints of a diameter of circle are ....

(a) equal in length. (b) parallel. (c) intersection. (d) perpendicular.

---

8 The number of circles that can be drawn passing through the endpoints of a line segment is ....

(a) one. (b) two.

(c) three. (d) an infinite number.

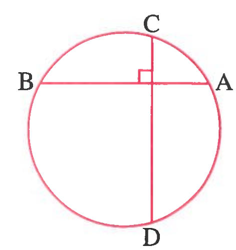
---

9 A rectangle of length 5 cm. and width 3 cm. , then its area = ... cm^2.

(a) 4 (b) 8 (c) 15 (d) 16

---

10 **In the opposite figure :**



$\overline{AB} \perp \overline{CD}$

, then m $(\overset{\frown}{AC})$ + m $(\overset{\frown}{BD})$ = ...

(a) 45° (b) 90°

(c) 180° (d) 270°

---

11 We can draw a circle passes through the vertices of ....

(a) trapezium. (b) parallelogram. (c) rhombus. (d) rectangle.

---

12 The number of common tangents for two circles touching internally equals ....

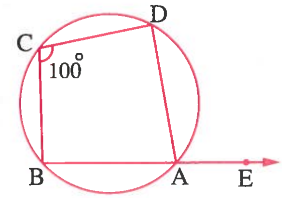
(a) zero. (b) one. (c) two. (d) three.

---

13 If M and N are two centres of two circles touching externally their radii lengths are 5 cm. , 9 cm. , then MN = ... cm.

(a) 14 (b) 4 (c) 5 (d) 9

14 **In the opposite figure :**



m (∠ C) = 100°

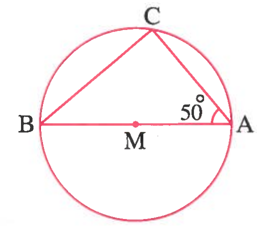
, then m (∠ DAE) = ...

(a) 80° (b) 100°

(c) 50° (d) 90°

---

15 **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M , m (∠ A) = 50°

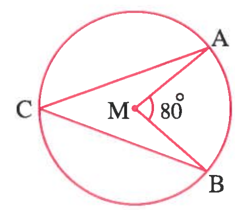
, then m (∠ ABC) = ...

(a) 25° (b) 40°

(c) 50° (d) 30°

---

16 **In the opposite figure :**



m (∠ AMB) = 80°

, then m (∠ ACB) = ...

(a) 40° (b) 80°

(c) 60° (d) 20°

---

17 The number of axes of symmetry of the equilateral triangle is ....

(a) zero. (b) one. (c) two. (d) three.

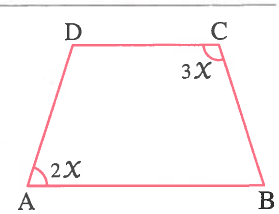
---

18 The measure of the arc which is quarter circle is ....

(a) 60° (b) 90° (c) 120° (d) 180°

---

19 **In the opposite figure :**



ABCD is a cyclic quadrilateral in which m (∠ A) = 2 X

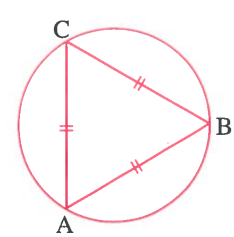
, m (∠ C) = 3 X , then X = ...

(a) 20° (b) 30°

(c) 32° (d) 36°

---

20 **In the opposite figure :**



Δ ABC is an equilateral triangle

, the length of $\overset{\frown}{AB}$ = 8 cm.

, then the circumference of its circumcircle = ... cm.

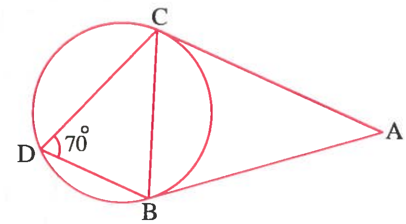
(a) 24 (b) 48

(c) 16 (d) 40

21 **In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle from A , m ($\angle$ D) = 70°

, then m ($\angle$ A) = ...



(a) 35°   (b) 70°

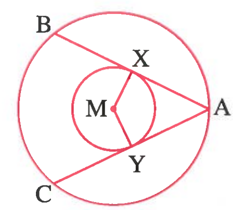
(c) 40°   (d) 20°

Second Essay questions

22 **In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the smaller circle M

Show that : AB = AC



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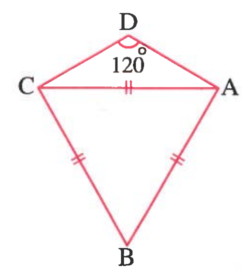
23 **In the opposite figure :**

m ($\angle$ D) = 120°

, Δ ABC is an equilateral triangle

Show that :

ABCD is cyclic quadrilateral



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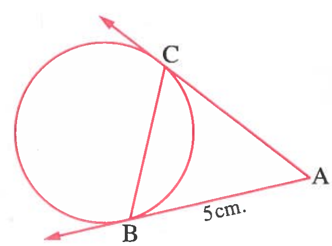
24 **In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents

, m $(\overset{\frown}{BC})$ = 120° , AB = 5 cm.

Find the perimeter of Δ ABC



Answers of governorates' examinations of geometry

1 Cairo

1

1 c   2 a   3 d   4 b   5 b   6 a

2

[a] $\because \overleftrightarrow{AD}$ is a tangent to the circle at D

$\therefore \overline{MD} \perp \overleftrightarrow{AD}$ $\therefore m(\angle ADM) = 90°$

, $\because$ E is the midpoint of $\overline{BC}$

$\therefore \overline{ME} \perp \overline{BC}$ $\therefore m(\angle BEM) = 90°$

From the quadrilateral ADME

$\therefore m(\angle DME) = 360° - (90° + 90° + 60°) = 120°$ (The req.)

[b] $\because \overleftrightarrow{CA}$, $\overleftrightarrow{CD}$ are two tangents to the circle M

$\therefore CA = CD$ (1)

, $\because \overleftrightarrow{CD}$ and $\overleftrightarrow{CB}$ are two tangents to the circle N

$\therefore CD = CB$ (2)

From (1) and (2) : $\therefore CA = CB$

$\therefore$ C is the midpoint of $\overline{AB}$ (Q.E.D.)

3

[a] In ΔABC :

$\because \overline{BA} \perp \overline{AC}$ $\therefore m(\angle BAC) = 90°$

$\therefore m(\angle C) = 180° - (90° + 40°) = 50°$

$\therefore m(\angle C) = m(\angle DAB)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle that passes through the vertices of the triangle ABC (Q.E.D.)

[b] $m(\angle A) = \frac{1}{2}\left[m(\overset{\frown}{CE}) - m(\overset{\frown}{BD})\right]$

$= \frac{1}{2}\left[100° - 30°\right] = 35°$ (The req.)

4

[a] In ΔBCM :

$\because BM = CM = r$

$\therefore m(\angle MCB) = m(\angle MBC) = 50°$

$\therefore m(\angle BMC) = 180° - (50° + 50°)$

$= 80°$ (First req.)

$\therefore m(\angle BAC) = \frac{1}{2} m(\angle BMC) = \frac{1}{2} \times 80° = 40°$

(inscribed and central angles subtendend by $\overset{\frown}{BC}$)

(Second req.)

[b] $\because \Delta ABD$ is an equilateral triangle

$\therefore m(\angle A) = 60°$

, $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle A) + m(\angle C) = 180°$

$\therefore m(\angle C) = 180° - 60° = 120°$ (The req.)

5

[a] $\because \overleftrightarrow{AO} \,/\!/\, \overline{ED}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore m(\angle OAD) = m(\angle ADE)$ (alternate angles)

, $\because m(\angle C)$ (inscribed) $= m(\angle OAB)$ (tangency)

$\therefore m(\angle C) = m(\angle ADE)$

$\therefore$ The figure DBCE is a cyclic quadrilateral (Q.E.D.)

[b] In ΔABC :

$\because m(\angle C) = 180° - (70° + 55°) = 55°$

$\therefore m(\angle B) = m(C)$ $\therefore AB = AC$

, $\because \overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

$\therefore MD = ME$ (Q.E.D.)

2 Giza

1

1 d   2 a   3 c   4 b   5 c   6 b

2

[a] In ΔBMC :

$\because MB = MC = r$

$\therefore m(\angle MCB) = m(\angle MBC) = 45°$

In ΔAMC : $\because MA = CM = r$

$\therefore m(\angle MCA) = m(\angle MAC) = 25°$

$\therefore m(\angle ACB) = 45° + 25° = 70°$

$\therefore m(\angle AMB) = 2\, m(\angle ACB) = 2 \times 70° = 140°$

(central and inscribed angles subtendend by $\overset{\frown}{AB}$)

[b] In ΔBCD : $\because CD = CB$

$\therefore m(\angle CBD) = m(\angle CDB) = 25°$

$\therefore m(\angle C) = 180° - 2 \times 25° = 130°$

$\therefore m(\angle A) + m(\angle C) = 50° + 130° = 180°$

$\therefore$ ABCD is a cyclic quadrilateral.

3

[a] $\because$ m ($\angle$ A) = $\frac{1}{2}$ [m ($\overset{\frown}{CD}$) – m ($\overset{\frown}{BE}$)]

$\therefore$ 35° = $\frac{1}{2}$ [110° – m ($\overset{\frown}{BE}$)]

$\therefore$ 70° = 110° – m ($\overset{\frown}{BE}$)

$\therefore$ m ($\overset{\frown}{BE}$) = 110° – 70° = 40° (The req.)

[b] $\because$ m ($\angle$ A) = m ($\angle$ D)

(two inscribed angles subtended by $\overset{\frown}{BC}$)

, $\because$ m ($\angle$ B) = m ($\angle$ C)

(two inscribed angles subtended by $\overset{\frown}{AD}$)

In Δ ECD : $\because$ ED = EC

$\therefore$ m ($\angle$ C) = m ($\angle$ D)

$\therefore$ m ($\angle$ A) = m ($\angle$ B)

$\therefore$ In Δ ABE : AE = BE (Q.E.D.)

4

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ BAC) = m ($\angle$ BDC) = 40°

(they are drawn on $\overline{BC}$ and on one side of it)

In Δ ABC :

$\therefore$ m ($\angle$ BAC) = m ($\angle$ ACB) = 40°

$\therefore$ AB = BC

$\therefore$ Δ ABC is isosceles. (Q.E.D.)

[b] $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle

$\therefore$ AB = AC

In Δ ABC : $\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB)

$= \frac{180° - 50°}{2} = 65°$

(First req.)

, $\because$ $\overline{AC}$ is a tangent-segment to the circle

$\therefore$ $\overline{MC} \perp \overline{AC}$ $\therefore$ m ($\angle$ MCA) = 90°

$\therefore$ m ($\angle$ MCB) = 90° – 65° = 25° (Second req.)

5

[a] $\because$ MD = ME = r

, $\because$ XE = YD

By subtracting : $\therefore$ MX = MY

, $\because$ X is the midpoint of $\overline{AB}$ $\therefore$ $\overline{MX} \perp \overline{AB}$

, $\because$ Y is the midpoint of $\overline{BC}$

$\therefore$ $\overline{MY} \perp \overline{BC}$ $\therefore$ AB = BC (Q.E.D.)

[b] $\because$ $\overleftrightarrow{AD}$ is a tangent to the circle at A

$\therefore$ m ($\angle$ DAC) (tangency) = m ($\angle$ B) (inscribed)

, $\because$ $\overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore$ m ($\angle$ AXY) = m ($\angle$ B) (corresponding angles)

$\therefore$ m ($\angle$ DAC) = m ($\angle$ AXY)

$\therefore$ $\overleftrightarrow{AD}$ is a tangent to the circle which passes through the vertices of Δ AXY (Q.E.D.)

3 Alexandria

1

1 c 2 d 3 a 4 b 5 c 6 d

2

[a] $\because$ AB = CD

, $\overline{ME} \perp \overline{AB}$, $\overline{MF} \perp \overline{CD}$

$\therefore$ MX = MY

, $\because$ ME = MF = r

By subtracting : $\therefore$ XE = YF (Q.E.D.)

[b] m ($\angle$ ACB) = $\frac{1}{2}$ m ($\angle$ BMA) = $\frac{1}{2}$ × 100° = 50°

(inscribed and central angles subtended by $\overset{\frown}{AB}$)

(First req.)

, In Δ AMB :

$\because$ AM = BM = r

$\therefore$ m ($\angle$ MAB) = m ($\angle$ MBA)

$= \frac{180° - 100°}{2} = 40°$ (Second req.)

3

[a] $\because$ m ($\angle$ A) = $\frac{1}{2}$ [m ($\overset{\frown}{EC}$) – m ($\overset{\frown}{BD}$)]

$\therefore$ 36° = $\frac{1}{2}$ [104° – m ($\overset{\frown}{BD}$)]

$\therefore$ 72° = 104° – m ($\overset{\frown}{BD}$)

$\therefore$ m ($\overset{\frown}{BD}$) = 104° – 72° = 32° (First req.)

, $\because$ m ($\overset{\frown}{DE}$) = m ($\overset{\frown}{BC}$)

$\therefore$ m ($\overset{\frown}{DE}$) = $\frac{360° - [104° + 32°]}{2}$ = 112°

(Second req.)

[b] $\because$ $\angle$ CDE is an exterior angle of the cyclic quadrilateral ABCD

$\therefore$ m ($\angle$ CDE) = m ($\angle$ ABC) = 120° (First req.)

, m ($\angle$ ADC) = 180° – 120° = 60°

$\because$ $\overline{AD}$ is a diameter in the circle M

$\therefore$ m ($\angle$ DCA) = 90°

From Δ ACD :

$\therefore$ m ($\angle$ CAD) = 180° – (90° + 60°) = 30°

(Second req.)

4

[a] $\because \overline{AB}$ is a tangent-segment to the circle M at A

$\therefore \overline{MA} \perp \overline{AB}$

$\therefore m(\angle BAM) = 90°$

$\therefore (MB)^2 = (AB)^2 + (AM)^2$

$= (8)^2 + (6)^2 = 100$

$\therefore MB = 10$ cm.

$, \because AM = DM = r = 6$ cm.

$\therefore BD = 10 - 6 = 4$ cm. (The req.)

[b] $\because \overrightarrow{AB}$ is tangent of the circle M at B

$\therefore \overline{MB} \perp \overrightarrow{AB}$ $\therefore m(\angle MBA) = 90°$

$, \because \overleftrightarrow{MN}$ is the line of centres

$, \overline{CD}$ is the common chord

$\therefore \overleftrightarrow{MN} \perp \overline{CD}$ $\therefore m(\angle MEC) = 90°$

$\therefore m(\angle MBA) + m(\angle MEA) = 90° + 90° = 180°$

$\therefore$ ABME is a cyclic quadrilateral (Q.E.D.)

5

[a] $\because \overline{AX}, \overline{AY}$ are two tangent-segments to the circle M

$\therefore AY = AX = 6$ cm (First req.)

$, \overline{MX} \perp \overline{AX}$ $\therefore m(\angle AXM) = 90°$

In Δ AXM :

$\therefore m(\angle XAM) = 180° - (90° + 65°) = 25°$

$, \because \overrightarrow{AM}$ bisects $\angle XAY$

$\therefore m(\angle XAY) = 2 \times 25° = 50°$ (Second req.)

[b] In Δ ABD :

$\because m(\angle ABD) = 180° - (75° + 50°) = 55°$

$\therefore m(\angle CBD) = 130° - 55° = 75°$

$\therefore m(\angle A) = m(\angle CBD)$

$\therefore \overline{BC}$ is a tangent-segment to the circle which passes through the points A , B and D(Q.E.D.)

4 El-Kalyoubia

1

1 d 2 b 3 a 4 b 5 c 6 a

2

[a] In the greater circle :

$\because \overline{ME} \perp \overline{AB}$

$\therefore$ E is the midpoint of $\overline{AB}$

$\therefore AE = EB$ (1)

In the smaller circle :

$\because \overline{ME} \perp \overline{CD}$

$\therefore$ E is the midpoint of $\overline{CD}$

$\therefore CE = DE$ (2)

subtracting (2) from (1) :

$\therefore AC = BD$ (Q.E.D.)

[b] $\because \overline{AB} \cap \overline{CD} = \{M\}$

$\therefore m(\angle AMC) = m(\angle DMB) = 25°$ (V.O.A)

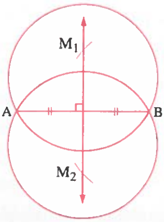
$\therefore m(\widehat{AC}) = 25°$

$\because \overline{CE} \,//\, \overline{AB}$

$\therefore m(\widehat{BE}) = m(\widehat{AC}) = 25°$ (The req.)

3

[a]



the number of solutions = 2

[b] $\because \overline{BC}$ is a diameter in the circle M

$\therefore m(\angle A) = 90°$

$, \because \overline{ED} \perp \overline{BC}$

$\therefore m(\angle BDE) = 90°$

$, \because m(\angle A) + m(\angle BDE) = 90° + 90°$ 180°

$\therefore$ ABDE is a cyclic quadrilateral (Q.E.D.1)

$\because \angle CED$ is an exterior angle of the cyclic quadrilateral ABDE

$\therefore m(\angle CED) = m(\angle B)$

$, \because m(\angle B) = \frac{1}{2} m(\widehat{AC})$

$\therefore m(\angle CED) = \frac{1}{2} m(\widehat{AC})$ (Q.E.D.2)

4

[a] $\because m(\angle ACB) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles subtendend by $\widehat{AB}$)

$\therefore m(\angle ACB) = \frac{1}{2} \times 120° = 60°$ (1)

$, \because \overleftrightarrow{CD} \,//\, \overline{AB}$ $\therefore m(\widehat{CA}) = m(\widehat{CB})$

$\therefore CA = CB$ (2)

From (1) and (2) :

$\therefore \Delta$ ABC is an equilateral triangle (Q.E.D.)

[b] $\because \overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M

$\therefore AB = AC$

$\therefore 2x + 1 = x + 4 \quad \therefore x = 3$ (First req.)

$\therefore AB = AC = 7$ cm. , $BC = 5$ cm.

$\therefore$ The perimeter of $\Delta ABC = 7 + 7 + 5 = 19$ cm. (Second req.)

5

[a] $\because m(\angle A) = \frac{1}{2}[m(\widehat{CE}) - m(\widehat{BD})]$

$\therefore 30° = \frac{1}{2}[120° - m(\widehat{BD})]$

$\therefore 60° = 120° - m(\widehat{BD})$

$\therefore m(\widehat{BD}) = 120° - 60° = 60°$ (First req.)

, $\because m(\widehat{DE}) = m(\widehat{BC})$

by adding $m(\widehat{BD})$ to both sides

$\therefore m(\widehat{BE}) = m(\widehat{DC})$

$\therefore m(\angle C) = m(\angle E)$

In ΔAEC : $\therefore AE = AC$ (Second req.)

[b] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle EDC) = m(\angle B)$ (1)

$\because \overrightarrow{EO} \parallel \overline{CD}$, $\overrightarrow{AE}$ is a transversal

$\therefore m(\angle DEO) = m(\angle EDC)$ (alternate angles) (2)

From (1) and (2) :

$\therefore m(\angle B) = m(\angle AEO)$

$\therefore \overrightarrow{EO}$ is a tangent to the circle which passes through the vertices of ΔEBA (Q.E.D.)

5 El-Sharkia

1

1 d   2 a   3 C   4 d   5 b   6 a

2

[a] $\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$

, $\because \overline{MH} \perp \overline{AC}$, $AB = AC$

$\therefore MD = MH$

$\because MX = MY = r$

By subtracting : $\therefore DX = HY$ (Q.E.D.)

[b] $\because \overline{AB} \parallel \overline{CD}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle BAC) + m(\angle ACD) = 180°$

(two interior angles in the same side of the transversal)

$\therefore m(\angle ACD) = 180° - 100° = 80°$

$\therefore m(\angle AMD) = 2\, m(\angle ACD) = 160°$

(central and inscribed angles subtended by $\widehat{AD}$) (The req.)

3

[a] $\Delta\Delta$ AHD , AHC :

in them $\begin{cases} AD = AC \\ \overline{AH} \text{ is a common side} \\ m(\angle DAH) = m(\angle CAH) \end{cases}$

$\therefore \Delta AHD \equiv \Delta AHC$

$\therefore m(\angle ADH) = m(\angle C)$

, $\because m(\angle C) = m(\angle O)$

(two inscribed angles subtended by $\widehat{AB}$)

$\therefore m(\angle O) = m(\angle ADH)$

$\therefore$ HDBO is a cyclic quadrilateral (Q.E.D.)

[b] $\because BX = CY$

$\therefore m(\widehat{BX}) = m(\widehat{CY})$

Adding $m(\widehat{XY})$ to both sides

$\therefore m(\widehat{BY}) = m(\widehat{CX})$

$\therefore m(\angle C) = m(\angle B)$

$\therefore$ In ΔABC : $AB = AC$

$\because BX = CY$

By subtracting : $\therefore AX = AY$ (Q.E.D.)

4

[a] $\because \overline{HA}$, $\overline{HC}$ are two tangent-segments of the circle

$\therefore HA = HC$

$\therefore$ In ΔAHC :

$m(\angle HAC) = m(\angle HCA)$

$= \frac{180° - 80°}{2} = 50°$

, $\because$ ACBD is a cyclic quadrilateral

$\therefore m(\angle D) + m(\angle ACB) = 180°$

$\therefore m(\angle ACB) = 180° - 130° = 50°$ (1)

, $\because m(\angle ABC)$ (inscribed)

$= m(\angle CAH)$ (tangency) $= 50°$ (2)

From (1) and (2) :

$\therefore$ In Δ ABC : m ($\angle$ ACB) = m ($\angle$ ABC)

$\therefore$ AB = CA (Q.E.D.1)

$\therefore$ m ($\angle$ BAC) = 180° – (50° + 50°) = 80°

$\therefore$ m ($\angle$ H) = m ($\angle$ BAC)

$\therefore$ $\overrightarrow{AB}$ is a tangent to the circle which passing through the points A , C and H (Q.E.D.2)

[b] $\because$ D is the midpoint of $\overline{XY}$

$\therefore$ $\overline{MD} \perp \overline{XY}$ $\therefore$ m ($\angle$ MDX) = 90°

$\because$ H is the midpoint of $\overline{XZ}$

$\therefore$ $\overline{MH} \perp \overline{XZ}$ $\therefore$ m ($\angle$ MHX) = 90°

From the quadrilateral XHMD

$\therefore$ m ($\angle$ HMD) = 360° – (100° + 90° + 90°) = 80°

$\therefore$ m ($\angle$ AMB) = m ($\angle$ HMD) = 80° (V.O.A)

In Δ AMB :

$\because$ AM = BM = r

$\therefore$ m ($\angle$ A) = m ($\angle$ B) = $\frac{180° - 80°}{2}$ = 50°

$\therefore$ m ($\angle$ AMB) > m ($\angle$ A) $\therefore$ AB > MB

$\therefore$ AB > r (Q.E.D.)

5

[a] $\because$ $\overrightarrow{AD}$, $\overrightarrow{AC}$ are two tangents to the circle M

$\therefore$ AD = AC (1)

, $\because$ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle N

$\therefore$ AB = AC (2)

From (1) and (2) :

$\therefore$ AB = AD = AC (First req.)

$\therefore$ The point A is the center of the circle passing through the points B , C , D

$\therefore$ m ($\angle$ BDC) = $\frac{1}{2}$ m ($\angle$ CAB) = $\frac{1}{2}$ × 68° = 34°

(inscribed and central angles subtended by $\overset{\frown}{BC}$)

(Second req.)

[b] $\because$ AB = CD

$\therefore$ m ($\overset{\frown}{AB}$) = m ($\overset{\frown}{CD}$)

, $\because$ m ($\angle$ AHB) = $\frac{1}{2}$ [m ($\overset{\frown}{AB}$) + m ($\overset{\frown}{CD}$)]

$\therefore$ m ($\angle$ AHB) = $\frac{1}{2}$ [m ($\overset{\frown}{AB}$) + m ($\overset{\frown}{AB}$)]

$\therefore$ m ($\angle$ AHB) = m ($\overset{\frown}{AB}$)

, $\because$ m ($\angle$ AMB) = m ($\overset{\frown}{AB}$)

$\therefore$ m ($\angle$ AHB) = m ($\angle$ AMB)

and they are drawn on $\overline{AB}$ and on one side of it

$\therefore$ ABMH is a cyclic quadrilateral (First req.)

$\therefore$ m ($\angle$ AMH) = m ($\angle$ ABH) = 40°

$\therefore$ m ($\angle$ ADC) = m (ABC) = 40°

(two inscribed angles subtendend by $\overset{\frown}{AC}$)

(Second req.)

6 El-Monofia

1

1 b	2 c	3 c	4 a	5 d	6 a

2

[a] $\because$ $\overline{MY} \perp \overline{CD}$

$\therefore$ Y is the midpoint of $\overline{CD}$

$\therefore$ CD = 2 YD = 10 cm.

, $\because$ $\overline{MX} \perp \overline{AB}$, MX = MY

$\therefore$ AB = CD = 10 cm. (The req.)

[b] $\because$ $\overrightarrow{AD}$ is a tangent to the circle M

$\therefore$ $\overleftrightarrow{AD} \perp \overline{MD}$

$\therefore$ m ($\angle$ ADM) = 90°

, $\because$ E is a midpoint of $\overline{BC}$

$\therefore$ $\overline{ME} \perp \overline{BC}$

$\therefore$ m ($\angle$ AEM) = 90°

, $\because$ m ($\angle$ ADM) + m ($\angle$ AEM) = 90° + 90°
= 180°

$\therefore$ ADME is a cyclic quadriltateral (First req.)

, m ($\angle$ M) = 180° – 60° = 120° (Second req.)

3

[a] In Δ ABC :

$\because$ x° + 2 x° + 3 x° = 180°

$\therefore$ 6 x° = 180°

$\therefore$ x° = 30° (First req.)

$\therefore$ m ($\angle$ C) = m ($\angle$ BAD) = 30°

$\therefore$ $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , B and C (Second req.)

[b] $\because$ m ($\angle$ A) = $\frac{1}{2}$ [m ($\overset{\frown}{BC}$) – m ($\overset{\frown}{DE}$)]

$\therefore$ 20° = $\frac{1}{2}$ [90° – m ($\overset{\frown}{DE}$)]

$\therefore$ 40° = 90° – m ($\overset{\frown}{DE}$)

$\therefore$ m ($\overset{\frown}{DE}$) = 90° – 40° = 50°

$\because$ $\overline{CE}$ is a diameter.

$\therefore$ m ($\overset{\frown}{CE}$) = 180°

$\therefore$ m ($\overset{\frown}{DB}$) = 180° – (90° + 50°)
= 40° (The req.)

4

[a] $\because \overleftrightarrow{AB}$ is a tangent to the circle

$\therefore$ m ($\angle$ ABC) (tangency)

= m ($\angle$ D) (inscribed) = 65°

, $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore$ AB = AC

$\therefore$ In Δ ABC : m ($\angle$ ABC) = m ($\angle$ ACB) = 65°

$\therefore$ m ($\angle$ BAC) = 180° – (65° + 65°) = 50°

(The req.)

[b] $\because$ m ($\angle$ BMC) = 2 m ($\angle$ A)

= 2 × 30° = 60°

(central and inscribed angles subtendend by $\overset{\frown}{BC}$)

In Δ BMC :

$\because$ MB = MC = r

$\therefore$ Δ BMC is equilateral

$\therefore$ MB = CM = BC = r = 7 cm.

$\therefore$ the circumference of the circle

M = 2 π r = 2 × $\frac{22}{7}$ × 7 = 44 cm. (The req.)

5

[a] In Δ ABD :

$\because$ AB = AD $\therefore$ m ($\angle$ ADB) = m ($\angle$ ABD) = 35°

$\therefore$ m ($\angle$ A) = 180° – (35° + 35°) = 110°

, $\because$ m ($\angle$ A) + m ($\angle$ C) = 110° + 70° = 180°

$\therefore$ ABCD is a cyclic quadrilateral (Q.E.D.)

[b] $\because \overline{AB}$ is a tangent-segment to the circle at A

$\therefore \overline{AM} \perp \overline{AB}$ $\therefore$ m ($\angle$ BAM) = 90°

In Δ ABM :

$\therefore (BM)^2 = (AB)^2 + (AM)^2$

$= 12^2 + 9^2 = 225$

$\therefore$ BM = $\sqrt{225}$ = 15 cm.

, $\because$ MC = MA = r = 9 cm.

$\therefore$ BC = 15 – 9 = 6 cm. (The req.)

7 El-Gharbia

1

1 b	2 c	3 b	4 d	5 d	6 a

2

[a] $\because \overline{BC}$ is a diameter in the circle M

$\therefore$ m ($\angle$ A) = 90°

, $\because \overline{DE} \perp \overline{BC}$

$\therefore$ m ($\angle$ BED) = 90°

, $\because$ m ($\angle$ A) + m ($\angle$ BED) = 90° + 90° = 180°

$\therefore$ ABED is a cyclic quadrilateral. (Q.E.D.1)

, $\because$ $\angle$ EDC is an exterior angle of the cyclic quadrilateral ABED

$\therefore$ m ($\angle$ EDC) = m ($\angle$ B)

, $\because$ m ($\angle$ B) = $\frac{1}{2}$ m ($\overset{\frown}{AC}$)

$\therefore$ m ($\angle$ EDC) = $\frac{1}{2}$ m ($\overset{\frown}{AC}$) (Q.E.D.2)

[b] $\because$ m ($\overset{\frown}{BD}$) = 2 m ($\angle$ C) = 2 × 70° = 140°

, $\because$ AB = AD

$\therefore$ m ($\overset{\frown}{AB}$) = m ($\overset{\frown}{AD}$) = 140° ÷ 2 = 70°

$\because \overline{BC}$ is a diameter in the circle M

$\therefore$ m ($\overset{\frown}{BC}$) = 180°

$\therefore$ m ($\overset{\frown}{ABC}$) = m ($\overset{\frown}{AB}$) + m ($\overset{\frown}{BC}$)

= 70° + 180° = 250°

$\therefore$ m ($\angle$ ADC) = $\frac{1}{2}$ m ($\overset{\frown}{ABC}$)

= $\frac{1}{2}$ × 250° = 125° (The req.)

3

[a] $\because$ m ($\angle$ A) = $\frac{1}{2}$ [m ($\overset{\frown}{EC}$) – m ($\overset{\frown}{BD}$)]

$\therefore$ 36° = $\frac{1}{2}$ [154° – m ($\overset{\frown}{BD}$)]

$\therefore$ 72° = 154° – m ($\overset{\frown}{BD}$)

$\therefore$ m ($\overset{\frown}{BD}$) = 154° – 72° = 82°

, $\because$ m ($\overset{\frown}{DE}$) = m ($\overset{\frown}{BC}$)

$\therefore$ m ($\overset{\frown}{DE}$) = $\frac{360° - (154° + 82°)}{2}$ = 62°

(The req.)

[b] $\because \overline{XY}$ // $\overline{BC}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore$ m ($\angle$ AXY) = m ($\angle$ B) (corresponding angles)

, $\because$ m ($\angle$ B) (inscribed) = m ($\angle$ CAD) (tangency)

$\therefore$ m ($\angle$ AXY) = m ($\angle$ CAD)

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

4

[a] $\because \overrightarrow{XA}$, $\overrightarrow{XB}$ are two tangents to the circle

$\therefore$ XA = XB

In Δ XAB :

$\therefore$ m ($\angle$ XAB) = m ($\angle$ XBA) = $\frac{180° - 70°}{2}$ = 55°

, $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ BAD) + m ($\angle$ C) = 180°

$\therefore$ m ($\angle$ BAD) = 180° – 125° = 55°

$\therefore$ m ($\angle$ XAB) = m ($\angle$ BAD)

$\therefore$ $\overrightarrow{AB}$ bisects $\angle$ DAX (Q.E.D.1)

, $\because$ m ($\angle$ XBA) = m ($\angle$ BAD) = 55°

and they are alternate angles

$\therefore$ $\overleftrightarrow{AD}$ // $\overrightarrow{XB}$ (Q.E.D.2)

[b] m ($\angle$ AMB) = 180° – 30° = 150°

$\therefore$ m ($\angle$ ACB) = $\frac{1}{2}$ m ($\angle$ AMB) = $\frac{1}{2}$ × 150° = 75°

(inscribed and central angles subtended by $\overset{\frown}{AB}$)

(The req.)

5

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MX} \perp \overline{AB}$

, $\because$ $\overline{MY} \perp \overline{CD}$, AB = CD

$\therefore$ MX = MY

, $\because$ ME = MF = r

By subtracting : $\therefore$ EX = FY (Q.E.D.)

[b] In Δ AMC : $\because$ MA = MC = r

$\therefore$ m ($\angle$ CAM) = m ($\angle$ ACM) = 50°

$\therefore$ m ($\angle$ M) = 180° – 2 × 50° = 80°

$\therefore$ m ($\angle$ B) = $\frac{1}{2}$ m ($\angle$ M) = 40°

(inscribed and central angles subtended by $\overset{\frown}{AC}$)

In Δ ABC : $\because$ AC = BC

$\therefore$ m ($\angle$ CAB) = m ($\angle$ B) = 40°

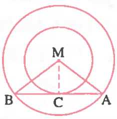
$\therefore$ m ($\angle$ MAB) = 50° – 40° = 10° (The req.)

8 El-Dakahlia

1

[a] 1 b 2 a 3 d

[b] Construction : Draw $\overline{MC}$



Proof : $\because$ $\overline{AB}$ touches the smaller circle at C

$\therefore$ $\overline{MC} \perp \overline{AB}$

$\therefore$ C is the midpoint of $\overline{AB}$

$\therefore$ AB = 2 AC

In Δ ACM : $\because$ m ($\angle$ ACM) = 90°

$\therefore$ AC = $\sqrt{5^2 - 3^2}$ = 4 cm.

$\therefore$ AB = 2 × 4 = 8 cm.

$\therefore$ The perimeter of Δ MAB = 5 + 5 + 8 = 18 cm.

(The req.)

2

[a] 1 a 2 d 3 c

[b] $\because$ The circle M $\cap$ the circle N = {A , B}

, $\overleftrightarrow{MN}$ is the line of centres

$\therefore$ $\overleftrightarrow{MN}$ is the axis of symmetry of $\overline{AB}$

, $\because$ C $\in \overleftrightarrow{MN}$

$\therefore$ AC = BC

, $\because$ $\overline{MX} \perp \overline{BC}$, $\overline{MY} \perp \overline{AC}$

$\therefore$ MX = MY (Q.E.D.)

3

[a] $\because$ CD = BE

$\therefore$ m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{BE}$)

Adding m ($\overset{\frown}{DE}$) to both sides

$\therefore$ m ($\overset{\frown}{CE}$) = m ($\overset{\frown}{BD}$)

$\therefore$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ In Δ ABC : AC = AB

$\because$ CD = BE

By subtracting : $\therefore$ AD = AE (Q.E.D.)

[b] $\because$ ABCD is a parallelogram

$\therefore$ m ($\angle$ A) = m ($\angle$ C) (1)

$\because$ DEBC is a cyclic quadrilateral and $\angle$ DEA is an exterior angle of it

$\therefore$ m ($\angle$ DEA) = m ($\angle$ C) (2)

From (1) and (2) :

$\therefore$ m ($\angle$ A) = m ($\angle$ DEA)

In Δ ADE :

$\therefore$ m ($\angle$ A) = m ($\angle$ DEA) = $\frac{180° - 30°}{2}$ = 75°

, $\because$ ABCD is a parallelogram

$\therefore$ m ($\angle$ B) + m ($\angle$ A) = 180°

$\therefore$ m ($\angle$ B) = 180° – 75° = 105° (The req.)

4

[a] $\because$ Δ ABC is equilateral $\therefore$ m ($\angle$ B) = 60°

$\therefore$ m ($\angle$ D) = m ($\angle$ B) = 60°

(two inscribed angles subtended by $\overset{\frown}{AC}$)

, $\because$ AD = DE

$\therefore$ Δ ADE is equilateral (Q.E.D.1)

, $\because$ $\angle$ AED is an exterior angle of Δ ACE

$\therefore$ m ($\angle$ AED) = m ($\angle$ EAC) + m ($\angle$ ACE)

, $\because$ m ($\angle$ ACB) = m ($\angle$ ACE) + m ($\angle$ DCB)

, $\because$ m ($\angle$ AED) = m ($\angle$ ACB) = 60°

$\therefore$ m ($\angle$ EAC) + m ($\angle$ ACE)

= m ($\angle$ ACE) + m ($\angle$ DCB)

$\therefore$ m ($\angle$ DCB) = m ($\angle$ EAC) (Q.E.D.2)

[b] $\because$ $\overline{XA}$, $\overline{YB}$ are two tangent-segments

, $\overline{AB}$ is a diameter

$\therefore$ $\overline{XA}$ // $\overline{YB}$, $\overline{AB} \perp \overline{XA}$, $\overline{YB} \perp \overline{AB}$

$\therefore$ ABYX is a trapezium

, $\because$ $\overline{XA}$, $\overline{XC}$ are two tangent-segments

$\therefore$ XA = XC

, $\because$ $\overline{YB}$, $\overline{YC}$ are two tangent-segments

$\therefore$ YB = YC

, $\because$ XC + YC = 13 cm. $\therefore$ XA + YB = 13 cm.

$\therefore$ The area of the trapezium AXYB

$= \frac{1}{2}$ [XA + YB] × AB $= \frac{1}{2} \times 13 \times 10 = 65 \text{ cm}^2$

(The req.)

5

[a] In the smaller circle

$\because$ m ($\angle$ XAB) (tangency)

= m ($\angle$ ADB) (inscribed) (1)

In the greater circle

$\because$ m ($\angle$ XAC) (tangency)

= m ($\angle$ AEC) (inscribed) (2)

From (1) and (2) :

$\therefore$ m ($\angle$ ADB) = m ($\angle$ AEC)

but they are corresponding angles

$\therefore$ $\overline{DB}$ // $\overline{EC}$ (Q.E.D.)

[b] $\because$ m ($\angle$ BDC) = m ($\angle$ BEC) = 90°

and they are drawn on $\overline{BC}$ and on one side of it

$\therefore$ DBCE is a cyclic quadrilateral.

$\therefore$ m ($\angle$ DBC) = m($\angle$ AED)

, $\because$ m ($\angle$ DBC) + m ($\angle$ DBX) = 90°

$\therefore$ m ($\angle$ AED) + m ($\angle$ DBX) = 90° (1)

, In Δ XBD :

$\because$ m ($\angle$ DXB) + m ($\angle$ DBX) = 90° (2)

From (1) and (2) :

$\therefore$ m ($\angle$ DXB) = m ($\angle$ AED)

$\therefore$ AXDE is a cyclic quadrilateral (Q.E.D.)

9 Suez

1

1 b 2 d 3 c 4 b 5 b 6 a

2

[a] In the greater circle :

$\because$ $\overline{MH} \perp \overline{AB}$

$\therefore$ H is the midpoint of $\overline{AB}$

$\therefore$ AH = HB (1)

In the smaller circle :

$\because$ $\overline{MH} \perp \overline{CD}$

$\therefore$ H is the midpoint of $\overline{CD}$

$\therefore$ CH = DH (2)

Subtracting (2) from (1) :

$\therefore$ AC = BD (Q.E.D.)

[b] $\because$ $\overline{AB}$ is a diameter in the circle M

$\therefore$ m ($\angle$ C) = 90°

, $\because$ $\overline{HD} \perp \overline{AB}$ $\therefore$ m ($\angle$ ADH) = 90°

, $\because$ m ($\angle$ C) + m ($\angle$ ADH) = 90° + 90° = 180°

$\therefore$ ACHD is a cyclic quadrilateral. (Q.E.D.)

3

[a] In Δ ABC :

$\because$ m ($\angle$ B) = m ($\angle$ C)

$\therefore$ AB = AC

, $\because$ Y is the midpoint of $\overline{AB}$

$\therefore$ $\overline{MY} \perp \overline{AB}$, $\because$ $\overline{MX} \perp \overline{AC}$

$\therefore$ MX = MY (Q.E.D.)

[b] m ($\angle$ AHC) $= \frac{1}{2}$ [m $(\overset{\frown}{AC})$ + m $(\overset{\frown}{BD})$]

$= \frac{1}{2}$ [50° + 110°]

$= \frac{1}{2} \times 160° = 80°$ (The req.)

4

[a] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments

$\therefore AB = AC$

$\therefore$ In Δ ABC :

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180° - 40°}{2} = 70°$

$\therefore m(\angle D)$ (inscribed) $= m(\angle ABC)$ (tangency) $= 70°$ (The req.)

[b] $\because m(\overset{\frown}{BC}) = m(\angle BMC) = 100°$

$\therefore m(\overset{\frown}{AB}) + m(\overset{\frown}{AC}) = 360° - 100° = 260°$

, $\because \overleftrightarrow{AD} \parallel \overline{BC}$

$\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{AC}) = \frac{260°}{2} = 130°$ (The req.)

5

[a] $\because \overline{AB}$ is a tangent-segment to the circle M

$\therefore \overline{MA} \perp \overline{AB}$ $\therefore m(\angle MAB) = 90°$

$\therefore MB = \sqrt{(AM)^2 + (AB)^2} = \sqrt{6^2 + 8^2} = 10$ cm.

$\because MA = MD = r = 6$ cm.

$\therefore BD = 10 - 6 = 4$ cm. (The req.)

[b] $\because$ ABCD is a cyclic quadrilateral.

$\therefore m(\angle A) + m(\angle C) = 180°$

$\therefore m(\angle A) = 180° - 130° = 50°$ (First req.)

$\therefore m(\angle BMD) = 2\, m(\angle A) = 2 \times 50° = 100°$

(central and inscribed angles subtended by $\overset{\frown}{BD}$)

(Second req.)

10 Damietta

1

1 a 2 c 3 b 4 d 5 a 6 c

2

[a] $\because \overline{AB}$ is a tangent-segment to the circle M

$\therefore \overline{MA} \perp \overline{AB}$

$\therefore m(\angle MAB) = 90°$

In Δ MAB : $\because m(\angle B) = 30°$

$\therefore MB = 2\, MA = 2 \times 7 = 14$ cm. (The req.)

[b] $m(\angle ADB) = \frac{1}{2} m(\overset{\frown}{AB}) = \frac{1}{2} \times 110° = 55°$

(First req.)

, $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle ADC) = m(\angle HBC) = 85°$

$\therefore m(\angle BDC) = 85° - 55° = 30°$

$\therefore m(\angle BAC) = m(\angle BDC) = 30°$

(two inscribed angles subtended by $\overset{\frown}{BC}$)

(Second req.)

3

[a] $\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\angle ACB) = 90°$

, $\because$ E is the midpoint of $\overline{HD}$

$\therefore \overline{ME} \perp \overline{HD}$

$\therefore m(\angle BEL) = 90°$

$\because m(\angle BEL) + m(\angle LCB) = 90° + 90° = 180°$

$\therefore$ LCBE is a cyclic quadrilateral (Q.E.D.)

[b] $\because \overline{MX} \perp \overline{AB}$ (1)

$\therefore m(\angle AXM) = 90°$

, $\because \overline{MY} \perp \overline{AC}$

$\therefore m(\angle AYM) = 90°$

From the quadrilateral AXMY

$\therefore m(\angle XMY) = 360° - (70° + 90° + 90°) = 110°$

(First req.)

$\because MD = MH = r$

, $XD = YH$ $\therefore MX = MY$

, $\because \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

$\therefore AB = AC$ (Second req.)

4

[a] $m(\angle ABH)$ (tangency) $= \frac{1}{2} m(\angle AMB)$ (central) $= \frac{1}{2} \times 130° = 65°$

(First req.)

, $\because m(\overset{\frown}{AB}) = m(\angle AMB) = 130°$

$\therefore m(\overset{\frown}{ADB}) = 360° - 130° = 230°$

(Second req.)

[b] $\because m(\angle LAN) = \frac{1}{2} m(\angle LMN)$

(inscribed and central angles subtendend by $\overset{\frown}{LN}$)

$\therefore m(\angle LAN) = \frac{1}{2} \times 120° = 60°$ (1)

, $\because \overleftrightarrow{AD} \parallel \overline{LN}$ $\therefore m(\overset{\frown}{AL}) = m(\overset{\frown}{AN})$

$\therefore AL = AN$ (2)

From (1) and (2) :

$\therefore$ The triangle ALN is an equilateral triangle

(Q.E.D.)

5

[a] $m(\angle AHC) = \frac{1}{2}[m(\overset{\frown}{AC}) + m(\overset{\frown}{DB})]$

$= \frac{1}{2}[50^\circ + 100^\circ]$

$= \frac{1}{2} \times 150^\circ = 75^\circ$ (The req.)

[b] $\because \overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle

$\therefore AB = AC$

In ΔABC :

$\therefore m(\angle ABC) = m(\angle ACB)$

$= \frac{180^\circ - 40^\circ}{2} = 70^\circ$

$\therefore m(\angle BDC)$ (inscribed) $= (\angle ABC)$ (tangency)

$= 70^\circ$ (First req.)

, $\because \overleftrightarrow{AB} \parallel \overline{CD}$, $\overleftrightarrow{BC}$ is a transversal

$\therefore m(\angle BCD) = m(\angle ABC) = 70^\circ$

(alternate angles)

In ΔBCD :

$\therefore m(\angle CBD) = 180^\circ - (70^\circ + 70^\circ) = 40^\circ$

$\therefore m(\angle CBD) = m(\angle BAC) = 40^\circ$

$\therefore \overline{BD}$ is a tangent-segment to the circumcircle of the triangle ABC (Q.E.D.)

11 Kafr El-Sheikh

1

1 b 2 d 3 b 4 b 5 b 6 b

2

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\therefore m(\angle MXB) = 90^\circ$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$ $\therefore m(\angle MYC) = 90^\circ$

, $\because AB = AC$ $\therefore MX = MY$

In ΔXYM : $\therefore m(\angle MXY) = m(\angle MYX)$

$\therefore m(\angle MXY) + m(\angle MXB)$

$= m(\angle MYX) + M(\angle MYC)$

$\therefore m(\angle YXB) = m(\angle XYC)$ (Q.E.D.)

[b] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle ADC) = m(\angle CBE) = 85^\circ$ (1)

, $\because m(\angle ADB) = \frac{1}{2} m(\overset{\frown}{AB})$

$= \frac{1}{2} \times 110^\circ = 55^\circ$ (2)

Subtracting (2) from (1) :

$\therefore m(\angle BDC) = 85^\circ - 55^\circ = 30^\circ$ (The req.)

3

[a] $\because \overrightarrow{AD}$ is a tangent to the circle M

$\therefore \overline{MD} \perp \overrightarrow{AD}$ $\therefore m(\angle ADM) = 90^\circ$

, $\because$ E in the midpoint of $\overline{BC}$

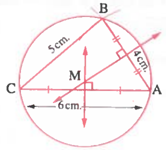
$\therefore \overline{ME} \perp \overline{BC}$ $\therefore m(\angle AEM) = 90^\circ$

From the quadrilateral ADME :

$\therefore m(\angle DME) = 360^\circ - [90^\circ + 90^\circ + 56^\circ] = 124^\circ$

(The req.)

[b]



ΔABC is an acute-angled triangle

4

[a] $\because \overleftrightarrow{BD}$ is a tangent to the circle at B

$\therefore m(\angle C)$ (inscribed) $= m(\angle ABD)$ (tangency)

(1)

, $\because \overline{XY} \parallel \overleftrightarrow{BD}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore m(\angle XBD) = m(\angle BXY)$ (2)

(alternate angles)

From (1) and (2) :

$\therefore m(\angle C) = m(\angle BXY)$

$\therefore$ AXYC is a cyclic quadrilateral. (Q.E.D.)

[b] In ΔABM : $\because AM = BM = r$

$\therefore m(\angle MBA) = m(\angle MAB) = 50^\circ$

$\therefore m(\angle AMB) = 180^\circ - (50^\circ + 50^\circ) = 80^\circ$

$\therefore m(\text{reflex } \angle AMB) = 360^\circ - 80^\circ = 280^\circ$

(First req.)

, $m(\angle C) = \frac{1}{2} m(\angle AMB) = \frac{1}{2} \times 80^\circ = 40^\circ$

(inscribed and central angles subtended by $\overset{\frown}{AB}$)

(Second req.)

5

[a] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents of the circle M

$\therefore AB = AC$

In ΔABC :

$\therefore m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$

(First req.)

$\because \overline{MC} \perp \overline{AC}$ $\therefore m(\angle ACM) = 90°$

$\therefore m(\angle MCB) = 90° - 70° = 20°$ (Second req.)

[b] $\because$ C is a midpoint of $\overline{AB}$

$\therefore \overline{MC} \perp \overline{AB}$ $\therefore m(\angle ACM) = 90°$

In Δ ACM :

$\therefore CM = \sqrt{(13)^2 - (12)^2} = 5$ cm.

$\therefore CD = 13 - 5 = 8$ cm.

$\therefore$ The area of $\Delta ADB = \frac{1}{2} AB \times CD$

$= \frac{1}{2} \times 24 \times 8 = 96 \text{ cm}^2$

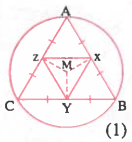
(The req.)

12 El-Fayoum

1

1 c 2 c 3 c 4 a 5 d 6 d

2



[a] Const : Draw

$\overline{MX}, \overline{MY}, \overline{MZ}$

$\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ (1)

, Y is the midpoint of $\overline{BC}$

$\therefore \overline{MY} \perp \overline{BC}$ (2)

, Z is the midpoint of $\overline{AC}$

$\therefore \overline{MZ} \perp \overline{AC}$ (3)

$, \because AB = BC = AC$ (4)

From (1) , (2) , (3) and (4) :

$\therefore MX = MY = MZ$

$\therefore$ M is the centre of the circumcircle of the triangle XYZ (Q.E.D.)

[b] $m(\angle AMB)$ (central)

$= 2\, m(\angle ABC)$ (tangency)

$= 2 \times 50° = 100°$ (The req.)

3

[a] $\because \overline{AB}$ is a diameter in the circle M

$\therefore m(\angle ADB) = 90°$

$\therefore m(\angle BDC) = 120° - 90° = 30°$

In Δ BCD : $\because$ CD = CB

$\therefore m(\angle DBC) = m(\angle BDC) = 30°$ (First req.)

$\therefore m(\angle C) = 180° - (30° + 30°) = 120°$

$, \because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle BAD) + m(\angle C) = 180°$

$\therefore m(\angle BAD) = 180° - 120° = 60°$ (Second req.)

[b] $m(\angle A) = \frac{1}{2}[m(\overset{\frown}{BC}) - m(\overset{\frown}{HD})]$

$= \frac{1}{2}[80° - 30°] = 25°$ (The req.)

4

[a] $\because \overline{AX}, \overline{AY}$ are two tangent-segments to the circle M

$\therefore AX = AY = 6$ cm. (1)

$, \because \overline{BX}, \overline{BZ}$ are two tangent-segments to the circle M

$\therefore BZ = BX = 2$ cm. (2)

$, \because \overline{CY}, \overline{CZ}$ are two tangent-segments to the circle M

$\therefore CY = CZ = 4$ cm. (3)

In Δ ABC : $\because (AC)^2 = (10)^2 = 100$

$, (AB)^2 + (BC)^2 = (8)^2 + (6)^2 = 100$

$\therefore (AC)^2 = (AB)^2 + (BC)^2$

$\therefore m(\angle B) = 90°$

$\therefore$ The area of $\Delta ABC = \frac{1}{2} AB \times BC$

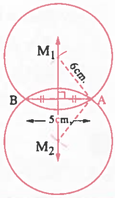
$= \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$

(First req.)

, the perimeter of Δ ABC

$= 8 + 6 + 10 = 24$ cm. (Second req.)

[b] We can draw two circles.



5

[a] $\because m(\overset{\frown}{AC}) = m(\overset{\frown}{CB}) = m(\overset{\frown}{BA})$

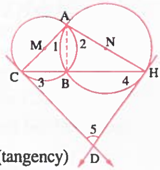
$= \frac{360°}{3} = 120°$

$\therefore m(\overset{\frown}{ACB}) = 120° + 120° = 240°$

$\therefore m(\angle ADB) = \frac{1}{2} m(\overset{\frown}{ACB}) = \frac{1}{2} \times 240° = 120°$

(The req.)

[b] Construction : Draw $\overline{AB}$



Proof : In the circle M :

$m(\angle 1)$ (inscribed)

$= m(\angle 3)$ (tangency)

In the circle N :

$m(\angle 2)$ (inscribed) $= m(\angle 4)$ (tangency)

In Δ DCH :

$\because$ m ($\angle$ 3) + m ($\angle$ 4) + m ($\angle$ 5) = 180°

$\therefore$ m ($\angle$ 1) + m ($\angle$ 2) + m ($\angle$ 5) = 180°

$\therefore$ m ($\angle$ HAC) + m ($\angle$ HDC) = 180°

$\therefore$ AHDC is a cyclic quadrilateral. (Q.E.D.)

13 Souhag

1

1 b 2 c 3 d 4 d 5 a 6 c

2

[a] $\because$ m ($\angle$ AHC) = $\frac{1}{2}$ [m $(\overset{\frown}{AC})$ – m $(\overset{\frown}{BD})$]

$\therefore$ 30° = $\frac{1}{2}$ [80° – m $(\overset{\frown}{BD})$]

$\therefore$ 60° = 80° – m $(\overset{\frown}{BD})$

$\therefore$ m $(\overset{\frown}{BD})$ = 80° – 60° = 20°

, $\because$ $\overline{AB}$ is a diameter $\therefore$ m $(\overset{\frown}{AB})$ = 180°

$\therefore$ m $(\overset{\frown}{CD})$ = 180° – (20° + 80°) = 80° (The req.)

[b] $\because$ D is midpoint of $\overline{AB}$

$\therefore$ $\overline{MD} \perp \overline{AB}$ $\therefore$ m ($\angle$ MDA) = 90°

, $\because$ $\overline{MH} \perp \overline{AC}$ $\therefore$ m ($\angle$ MHA) = 90°

From the quadrilateral ADMH :

$\therefore$ m ($\angle$ A) = 360° – (120° + 90° + 90°) = 60° (1)

, $\because$ MD = MH , $\overline{MD} \perp \overline{AB}$, $\overline{MH} \perp \overline{AC}$

$\therefore$ AB = AC (2)

From (1) , (2) :

$\therefore$ Δ ABC is an equilateral triangle. (Q.E.D.)

3

[a] $\because$ The two circles are touching internally at A

$\therefore$ MN = 10 – 6 = 4 cm. , $\overleftrightarrow{MN} \perp \overleftrightarrow{AB}$

, $\because$ The area of triangle BMN = $\frac{1}{2}$ × MN × AB

$\therefore$ 24 = $\frac{1}{2}$ × 4 × AB

$\therefore$ AB = $\frac{24}{2}$ = 12 cm. (The req.)

[b] $\because$ $\overrightarrow{AB}$ is a tangent to the circle M

$\therefore$ $\overline{MB} \perp \overrightarrow{AB}$

$\therefore$ m ($\angle$ ABM) = 90°

In Δ ABM :

$\therefore$ m ($\angle$ BMA) = 180° – (90° + 40°) = 50°

$\therefore$ m ($\angle$ BDC) = $\frac{1}{2}$ m ($\angle$ BMC) = $\frac{1}{2}$ × 50° = 25°

(inscribed and central angles subtended by $\overset{\frown}{BC}$)

(The req.)

4

[a] $\because$ $\overline{AB}$ is a diameter in circle M

$\therefore$ m ($\angle$ ACB) = 90°

, $\because$ $\overline{DH} \perp \overline{AB}$ $\therefore$ m ($\angle$ ADH) = 90°

$\therefore$ m ($\angle$ ACB) = m ($\angle$ ADH) = 90°

and they are drawn on $\overline{AH}$ and on one side of it.

$\therefore$ ACDH is a cyclic quadrilateral (Q.E.D.)

[b] $\because$ $\overline{AX}$, $\overline{AD}$ are two tangent-segments to the circle M

$\therefore$ AX = AD = 5 cm.

, $\because$ $\overline{BD}$, $\overline{BH}$ are two tangent-segments to the circle M

$\therefore$ BD = BH = 4 cm.

, $\because$ $\overline{CH}$, $\overline{CX}$ are two tangent-segments to the circle M

$\therefore$ CH = CX = 3 cm.

$\therefore$ The perimeter of Δ ABC = 5 + 5 + 4 + 4 + 3 + 3

= 24 cm. (The req.)

5

[a] $\because$ $\angle$ ABH is an exterior angle of the cyclic quadrilateral ABCD

$\therefore$ m ($\angle$ D) = m ($\angle$ ABH) = 100°

In Δ ACD :

$\therefore$ m ($\angle$ ACD) = 180° – (100° + 40°) = 40°

$\therefore$ m ($\angle$ ACD) = m ($\angle$ CAD)

$\therefore$ m $(\overset{\frown}{AD})$ = m $(\overset{\frown}{CD})$ (Q.E.D.)

[b] $\because$ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore$ AB = AC

In Δ ABC :

$\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB) = $\frac{180° - 50°}{2}$ = 65°

$\because$ m ($\angle$ BDC) (inscribed) = m ($\angle$ ABC) (tangency)

$\therefore$ m ($\angle$ BDC) = 65° (First req.)

$\because$ $\overrightarrow{AB}$ // $\overrightarrow{CD}$, $\overleftrightarrow{BC}$ is a transversal

$\therefore$ m ($\angle$ BCD) = m ($\angle$ ABC) = 65° (alternate angles)

$\therefore$ In Δ BCD :

m ($\angle$ DBC) = 180° – (65° + 65°) = 50°

$\therefore$ m ($\angle$ A) = m ($\angle$ DBC)

$\therefore$ $\overline{BD}$ is tangent-segment to the circle passing through the vertices of Δ ABC (Second req.)

14 Qena

1

1 b 2 c 3 b 4 a 5 d 6 d

2

[a] $\because$ $\overrightarrow{HA}$, $\overrightarrow{HC}$ are two tangents to the circle M

$\therefore$ HA = HC (1)

, $\because$ $\overrightarrow{HB}$, $\overrightarrow{HD}$ are two tangents to the circle N

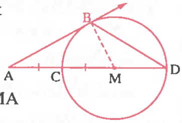
$\therefore$ HB = HD (2)

By Adding (1) and (2) :

$\therefore$ AB = CD (Q.E.D.)

[b] Constr : Draw $\overline{MB}$

Proof : $\because$ $\overrightarrow{AB}$ is a tangent



$\therefore$ $\overline{MB} \perp \overline{AB}$

$\therefore$ m ($\angle$ ABM) = 90°

In Δ AMB : $\because$ MB = $\frac{1}{2}$ MA

$\therefore$ m ($\angle$ A) = 30°

$\therefore$ m ($\angle$ AMB) = 180° – (90° + 30°) = 60°

$\therefore$ m ($\angle$ BDC) = $\frac{1}{2}$ m ($\angle$ BMC) = $\frac{1}{2}$ × 60° = 30°

(inscribed and central angles subtended by $\overset{\frown}{BC}$)

In Δ ABD :

$\therefore$ m ($\angle$ ABD) = 180° – (30° + 30°) = 120°

(The req.)

3

[a] $\because$ m ($\angle$ AEB) = 20°

$\therefore$ m $(\overset{\frown}{BD})$ = 40°

, $\because$ m ($\angle$ A) = $\frac{1}{2}$ [m $(\overset{\frown}{CE})$ – m $(\overset{\frown}{BD})$]

$\therefore$ 30° = $\frac{1}{2}$ [m $(\overset{\frown}{CE})$ – 40°]

$\therefore$ 60° = m $(\overset{\frown}{CE})$ – 40°

$\therefore$ m $(\overset{\frown}{CE})$ = 60° + 40° = 100°

$\therefore$ m ($\angle$ CDE) = $\frac{1}{2}$ m $(\overset{\frown}{CE})$

= $\frac{1}{2}$ × 100° = 50° (First req.)

$\therefore$ m ($\angle$ COE) = $\frac{1}{2}$ [m $(\overset{\frown}{BD})$ + m $(\overset{\frown}{CE})$]

= $\frac{1}{2}$ [40° + 100°] = 70°

(Second req.)

[b] In Δ ABD : $\because$ AB = AD

$\therefore$ m ($\angle$ ABD) = m ($\angle$ ADB) = 25°

$\therefore$ m ($\angle$ A) = 180° – (25° + 25°) = 130°

$\therefore$ m ($\angle$ A) = m ($\angle$ DCE)

$\therefore$ The figure ABCD is a cyclic quadrilateral

(Q.E.D.)

4

[a] $\because$ BCDE is a cyclic quadrilateral

$\therefore$ m ($\angle$ CBE) + m ($\angle$ D) = 180°

$\therefore$ m ($\angle$ CBE) = 180° – 125° = 55°

, $\because$ $\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore$ AB = AC

In Δ ABC : $\therefore$ m ($\angle$ ABC) = m ($\angle$ ACB)

$= \frac{180° - 70°}{2} = 55°$

$\therefore$ m ($\angle$ BEC) (inscribed)

= m ($\angle$ ACB) (tangency) = 55°

$\therefore$ m ($\angle$ CBE) = m ($\angle$ BEC) = 55°

$\therefore$ BC = EC

$\therefore$ Δ BCE is an isosceles triangle (Q.E.D.1)

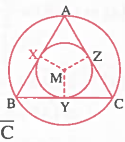
, $\because$ m ($\angle$ ACB) = m ($\angle$ CBE) = 55°

and they are alternate angles

$\therefore$ $\overrightarrow{AC}$ // $\overrightarrow{BE}$ (Q.E.D.2)

[b] Constr : Draw $\overline{MX}$, $\overline{MY}$, $\overline{MZ}$

Proof :



$\because$ $\overleftrightarrow{AB}$ is a tangent to

the smaller circle M

$\therefore$ $\overline{MX} \perp \overline{AB}$

, similary : $\overline{MY} \perp \overline{BC}$, $\overline{MZ} \perp \overline{AC}$

, $\because$ MX = MY = MZ = r in the smaller circle

$\therefore$ AB = BC = AC

$\therefore$ Δ ABC is an equilateral triangle (Q.E.D.)

5

[a] $\because$ m ($\angle$ CBD) = $\frac{1}{2}$ m ($\angle$ CMD)

(inscribed and central angles subtended by $\overset{\frown}{CD}$)

$\therefore$ m ($\angle$ CBD) = $\frac{1}{2}$ × 120° = 60°

, $\because$ $\angle$ CBD is an exterior angle of Δ ABD

$\therefore$ m ($\angle$ ADB) + m ($\angle$ A) = 60°

In Δ ABD : $\because$ AB = BD

$\therefore$ m ($\angle$ A) = m ($\angle$ ADB) = $\frac{60°}{2}$ = 30° (The req.)

[b] $\because$ $\angle$ BAE is an exterior angle of the cyclic quadrilateral ABCD

$\therefore$ m ($\angle$ C) = m ($\angle$ BAE) = 100°

In Δ BCD :

$\therefore$ m ($\angle$ BDC) = 180° – (100° + 40°) = 40°

$\therefore$ m ($\angle$ BDC) = m ($\angle$ CBD)

$\therefore$ m ($\overset{\frown}{BC}$) = m ($\overset{\frown}{CD}$) (Q.E.D.)

15 Aswan

1

1 c 2 c 3 b 4 a 5 d 6 b

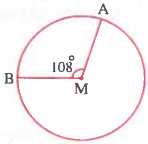
2

[a] $\because$ m ($\overset{\frown}{AB}$) = m ($\angle$ M) = 108°

$\therefore$ The length of $\overset{\frown}{AB}$

$= \frac{108°}{360°} \times 2 \times 3.14 \times 5$

= 9.42 cm.



(The req.)

[b] $\because$ $\overline{FL}$ // $\overleftrightarrow{XY}$

$\therefore$ m ($\overset{\frown}{XF}$) = m ($\overset{\frown}{XL}$)

$\therefore$ XF = XL (Q.E.D.)

3

[a] m ($\angle$ A) = $\frac{1}{2}$ [m ($\overset{\frown}{CE}$) – m ($\overset{\frown}{BD}$)]

$= \frac{1}{2}$ [100° – 30°] = 35° (The req.)

[b] $\because$ m ($\overset{\frown}{AB}$) : m ($\overset{\frown}{BC}$) : m ($\overset{\frown}{AC}$) = 3 : 4 : 5

$\therefore$ m ($\overset{\frown}{AB}$) = 3 x , m ($\overset{\frown}{BC}$) = 4 x , m ($\overset{\frown}{AC}$) = 5 x

$\therefore$ 3 x + 4 x + 5 x = 360°

$\therefore$ 12 x = 360°

$\therefore$ $x = \frac{360°}{12} = 30°$

$\therefore$ m ($\overset{\frown}{AC}$) = 5 × 30° = 150°

$\therefore$ m ($\angle$ ABC) = $\frac{1}{2}$ m ($\overset{\frown}{AC}$)

$= \frac{1}{2} \times 150° = 75°$ (The req.)

4

[a] $\because$ m ($\angle$ A) = m ($\angle$ C)

(two inscribed angles subtended by $\overset{\frown}{BD}$)

, $\because$ m ($\angle$ B) = m ($\angle$ D)

(two inscribed angles subtended by $\overset{\frown}{AC}$)

In Δ ADE : $\because$ EA = ED

$\therefore$ m ($\angle$ A) = m ($\angle$ D)

$\therefore$ m ($\angle$ B) = m ($\angle$ C)

In Δ EBC : $\therefore$ EB = EC (Q.E.D.)

[b] In Δ ACD :

$\because$ AD = DC

$\therefore$ m ($\angle$ CAD) = m ($\angle$ ACD) = 40°

$\therefore$ m ($\angle$ D) = 180° – (40° + 40°) = 100°

, $\because$ m ($\angle$ B) + m ($\angle$ D) = 80° + 100° = 180°

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

5

[a] $\because$ $\overrightarrow{XA}$, $\overrightarrow{XB}$ are two tangents to the circle

$\therefore$ XA = XB

In Δ XAB :

$\therefore$ m ($\angle$ XAB) = m ($\angle$ XBA) = $\frac{180° - 70°}{2}$ = 55° ,

, $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ BAD) + m ($\angle$ C) = 180°

$\therefore$ m ($\angle$ BAD) = 180° – 125° = 55°

, $\because$ m ($\angle$ XBA) = m ($\angle$ BAD) = 55°

and they are alternate angles

$\therefore$ $\overline{AD}$ // $\overrightarrow{XB}$ (Q.E.D.)

[b] $\because$ $\overline{NE}$ // $\overline{ZL}$, $\overleftrightarrow{XL}$ is a transversal

$\therefore$ m ($\angle$ XEN) = m ($\angle$ L) (corresponding angles)

, $\because$ m ($\angle$ L) (inscribed) = m ($\angle$ YXZ) (tangency)

$\therefore$ m ($\angle$ XEN) = m ($\angle$ YXN)

$\therefore$ $\overleftrightarrow{XY}$ is a tangent to the circle passing through the points X , N and E (Q.E.D.)

Answers of Port Said examinations of geometry

Exam 1 Port Said 2024

First Answers of multiple choice questions

1 (c) 2 (a) 3 (a) 4 (c) 5 (b)
6 (d) 7 (d) 8 (a) 9 (b) 10 (d)
11 (c) 12 (a) 13 (b) 14 (c) 15 (b)
16 (b) 17 (a) 18 (d) 19 (c) 20 (d)
21 (d)

Second Answers of essay questions

22

In the circle N :

$\because$ X is the midpoint of $\overline{AC}$

$\therefore \overline{NX} \perp \overline{AC}$

, $\because \overline{AB}$ is the common chord of the two circles M , N

, $\overleftrightarrow{MN}$ is the line of centres

$\therefore \overleftrightarrow{MN} \perp \overline{AB}$, $\because$ AC = AB

$\therefore$ NY = NX (Q.E.D.)

23

$\because \overrightarrow{AZ} \,//\, \overline{DH}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m ($\angle$ AHD) = m ($\angle$ HAZ) (alternate angles)

, $\because$ m ($\angle$ B) (inscribed) = m ($\angle$ CAZ) (tangency)

$\therefore$ m ($\angle$ B) = m ($\angle$ AHD)

$\therefore$ BCHD is a cyclic quadrilateral (Q.E.D.)

24

$\because \overline{BD}$ is a diameter

$\therefore$ m ($\angle$ BAD) = 90°

$\therefore$ In Δ ABD :

m ($\angle$ D) = 180° – (90° + 50°) = 40°

, $\because$ m ($\angle$ D) = m ($\angle$ BAC) = 40°

$\therefore \overleftrightarrow{AC}$ is a tangent to the circle M at A (Q.E.D.)

Exam 2 Port Said 2023

First Answers of multiple choice questions

1 (b) 2 (c) 3 (c) 4 (a) 5 (d)
6 (c) 7 (b) 8 (d) 9 (c) 10 (c)
11 (d) 12 (b) 13 (a) 14 (b) 15 (b)
16 (a) 17 (d) 18 (b) 19 (d) 20 (a)
21 (c)

Second Answers of essay questions

22

$\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the smaller circle

$\therefore \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

, $\because$ MX = MY = r

(lengths of two radii of the smaller circle)

$\therefore$ AB = AC (Q.E.D.)

23

$\because \Delta$ ABC is an equilateral triangle

$\therefore$ m ($\angle$ B) = m ($\angle$ BAC) = m ($\angle$ BCA) = 60°

$\therefore$ m ($\angle$ B) + m ($\angle$ D) = 60° + 120° = 180°

$\therefore$ ABCD is a cyclic quadrilateral (Q.E.D.)

24

$\because$ m ($\angle$ B) = $\frac{1}{2}$ m ($\overset{\frown}{BC}$) = $\frac{1}{2}$ × 120° = 60° (1)

, $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents.

$\therefore$ AB = AC (2)

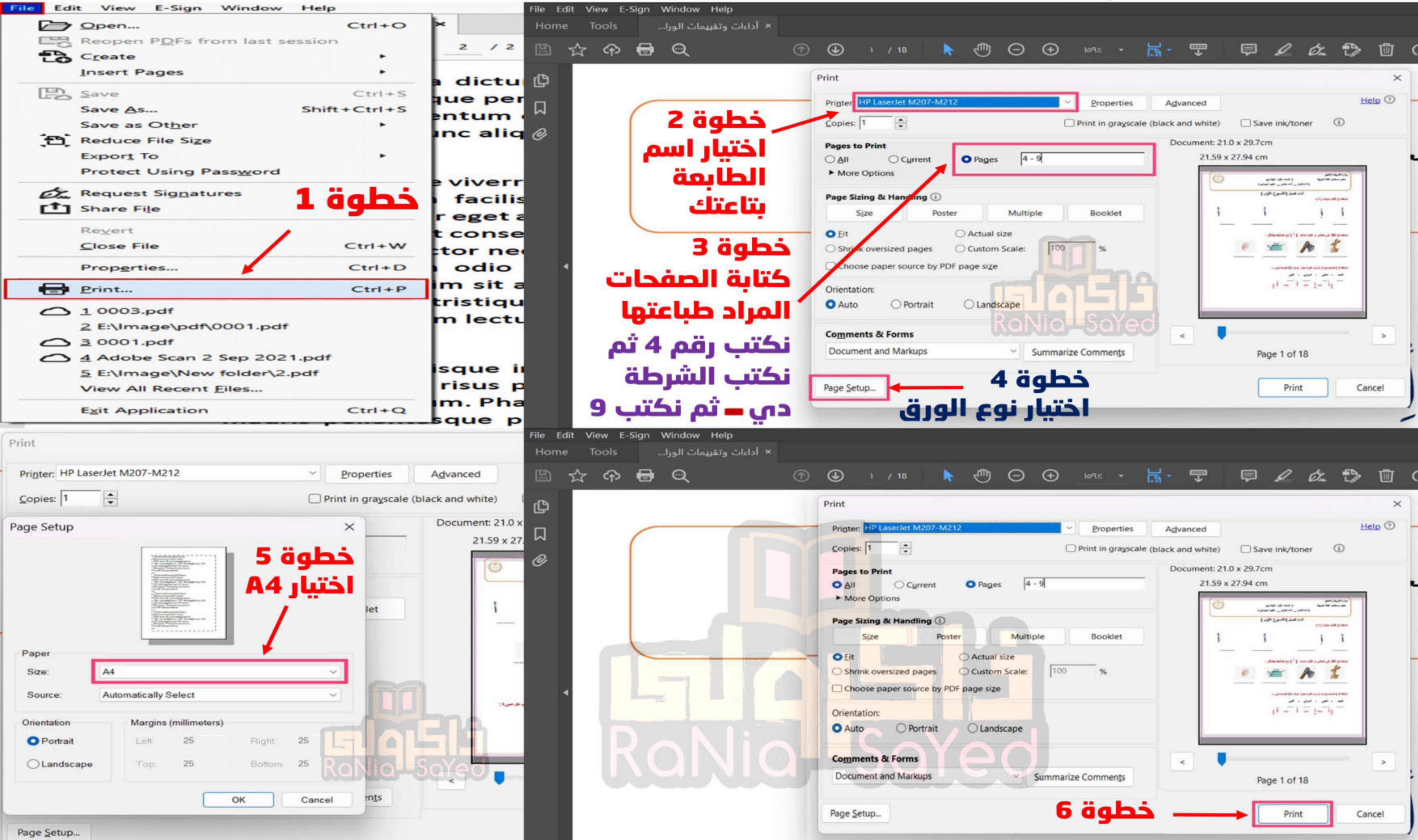
From (1) and (2) :

$\therefore \Delta$ ABC is an equilateral triangle.

$\therefore$ The perimeter of Δ ABC = 3 × 5 = 15 cm. (The req.)

كيفية طباعة صفحات معينة من ملف معين

مثلا ازاى نطبع الصفحات من صفحة 4 الى صفحة 9



امتحانات رقم (2)

الترم الثانى

1 Suez Governorate

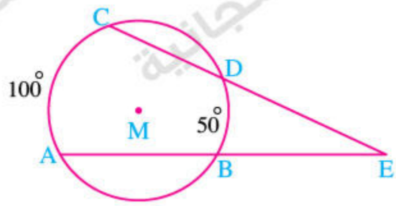
Answer the following questions : *(Calculator is permitted)*

1 Choose the correct answer from those given :

1. M and N are two circles , their radii lengths are 5 cm. and 3 cm. respectively , MN = 8 cm. , then the two circles are ……………
   (a) distant. (b) touching externally.
   (c) concentric. (d) touching internally.

2. The sum of measures of two supplementary angles equals ……………
   (a) 360° (b) 270° (c) 180° (d) 90°

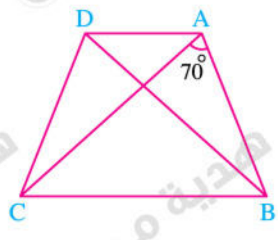
3. **In the opposite figure :**
   $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, m $(\overset{\frown}{AC}) = 100°$
   , m $(\overset{\frown}{DB}) = 50°$, then m (∠ E) = ……………



   (a) 25° (b) 75°
   (c) 50° (d) 30°

4. If the measures of two angles in a triangle are 40° and 100° , then the triangle is ……………
   (a) a scalene triangle. (b) an isosceles triangle.
   (c) an equilateral triangle. (d) a right-angled triangle.

5. **In the opposite figure :**
   ABCD is a cyclic quadrilateral
   , m (∠ BAC) = 70° , then m (∠ BDC) = ……………



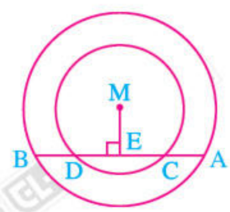
   (a) 35° (b) 70°
   (c) 140° (d) 80°

6. If $\overrightarrow{BD}$ bisects ∠ ABC , m (∠ ABC) = 60° , then m (∠ ABD) = ……………
   (a) 120° (b) 60° (c) 40° (d) 30°

---

2 [a] In the opposite figure :

Two concentric circles with centre M
, $\overline{AB}$ is a chord in the greater circle
and intersects the smaller at C , D , $\overline{ME} \perp \overline{AB}$
Prove that : AC = DB



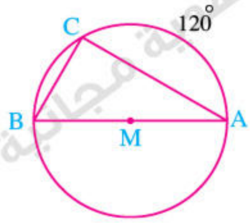
[b] In the opposite figure :

$\overline{AB}$ is a diameter in the circle M

, m $(\overset{\frown}{AC})$ = 120°

Find : 1 m (∠ C)

2 m (∠ A)



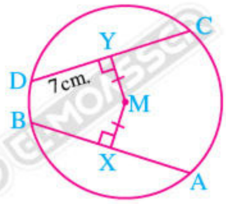
3 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords in the circle M

, $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

, MX = MY , YD = 7 cm.

Find : the length of $\overline{AB}$

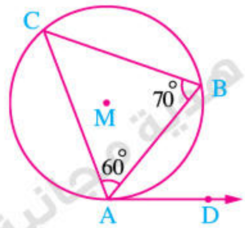


[b] In the opposite figure :

$\overrightarrow{AD}$ is a tangent to the circle M at A

, m (∠ ABC) = 70° , m (∠ BAC) = 60°

Find : m (∠ BAD)



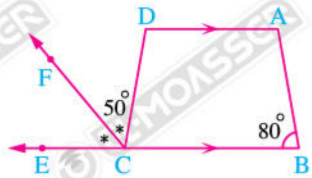
4 **[a]** State two cases of a cyclic quadrilateral.

[b] In the opposite figure :

$\overline{AD}$ // $\overrightarrow{BC}$, $\overrightarrow{CF}$ bisects ∠ DCE

, m (∠ DCF) = 50° , m (∠ B) = 80°

Prove that : ABCD is a cyclic quadrilateral.



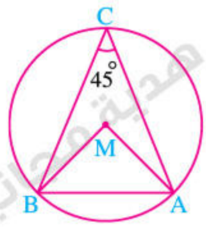
5 **[a] In the opposite figure :**

Δ ABC is inscribed in the circle M

, m (∠ ACB) = 45°

Find : 1 m (∠ AMB)

2 m (∠ MAB)

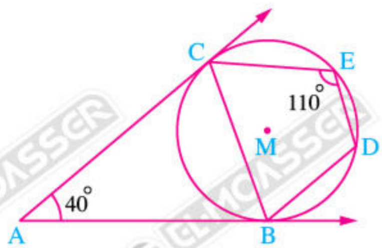


[b] In the opposite figure :

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C

, m (∠ E) = 110° , m (∠ A) = 40°

Prove that : m (∠ ABC) = m (∠ DBC)



2 Kafr El-Sheikh Governorate

Answer the following questions : *(Calculators are permitted)*

1 Choose the correct answer from those given :

1. The measure of the arc which equals one third of the measure of the circle equals ....
   (a) 60° (b) 120° (c) 180° (d) 270°

2. In the right-angled triangle , the length of the median drawn from the right angle equals .... the length of the hypotenuse.
   (a) fourth (b) one third (c) half (d) twice

3. M and N are two touching externally circles with radii lengths 9 cm. and 5 cm. , then MN = ... cm.
   (a) 5 (b) 9 (c) 4 (d) 14

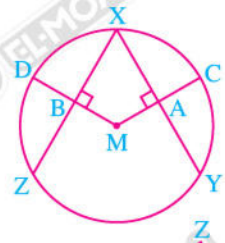
4. Δ ABC is a right-angled triangle at B , AB = 6 cm. , AC = 10 cm. , then BC = ... cm.
   (a) 6 (b) 8 (c) 10 (d) 4

5. If ABCD is a cyclic quadrilateral , m ($\angle$ B) = 70° , then m ($\angle$ D) = ...
   (a) 20° (b) 70° (c) 110° (d) 90°

6. If a straight line intersects two parallel straight lines , then each two alternate angles are ....
   (a) equal in measure. (b) supplementary. (c) complementary. (d) otherwise.

---

2 [a] In the opposite figure :

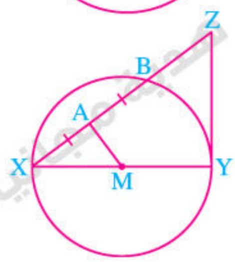


If XY = XZ , $\overrightarrow{MA} \perp \overline{XY}$

, $\overrightarrow{MB} \perp \overline{XZ}$

, **prove that :** AC = BD

[b] In the opposite figure :



$\overline{XY}$ is a diameter in a circle M

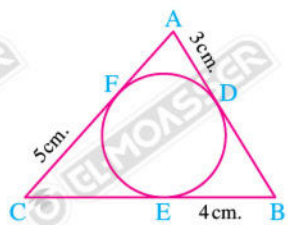
, $\overline{YZ}$ is a tangent-segment at Y

, A is the midpoint of $\overline{XB}$

Prove that : the figure MAZY is a cyclic quadrilateral.

---

3 [a] In the opposite figure :



A circle touches the sides of a triangle $\overline{AB}$

, $\overline{BC}$, $\overline{CA}$ at D , E , F respectively

, AD = 3 cm. , BE = 4 cm. , CF = 5 cm.

Find : the perimeter of Δ ABC

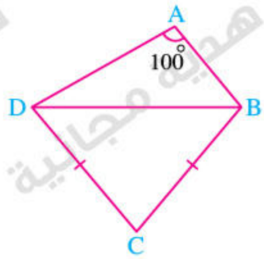
[b] In the opposite figure :

ABCD is a cyclic quadrilateral

, BC = DC

, m (∠ A) = 100°

Find : m (∠ CBD)



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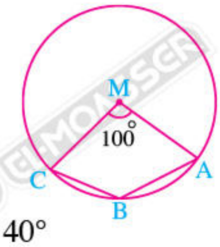
4 **[a] In the opposite figure :**

M is a circle , $\overline{MA}$ and $\overline{MC}$ are two radii

, m (∠ AMC) = 100° , B ∈ $\overset{\frown}{AC}$

Find : m (∠ ABC)



[b] $\overline{AB}$ is a diameter in a circle , draw the two chords $\overline{AC}$ and $\overline{BC}$, m (∠ A) = 40°

Find by proof : m (∠ B)

---

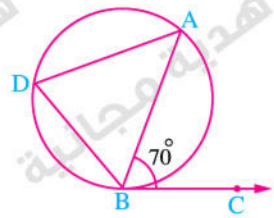
5 **[a] In the opposite figure :**

ABD is a triangle inscribed in a circle

, $\overrightarrow{BC}$ is a tangent to the circle at B , m (∠ ABC) = 70°

Find : 1 m (∠ D)

2 m ($\overset{\frown}{AB}$) "the minor arc"



[b] Using your geometric tools , draw $\overline{AB}$ with length 5 cm. , then draw on one figure a circle passing through the two points A and B and its diameter length is 5 cm. What are the possible solutions ?

3 El-Fayoum Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from the given answers :**

1 The circumference of a circle is 20 π cm. , then its area equals ·············· cm^2.

(a) 10 π (b) 100 π (c) 200 π (d) 400 π

2 The measure of the interior angle of the regular pentagon equals ··············

(a) 60° (b) 90° (c) 108° (d) 120°

3 Two circles M , N are touching internally , if their radii lengths are 5 cm. , 9 cm. , then MN = ·············· cm.

(a) 14 (b) 4 (c) 5 (d) 9

4 The measure of the inscribed angle is ·············· the measure of the central angle subtended by the same arc.

(a) third (b) quarter (c) half (d) twice

5 The measure of the exterior angle at any vertex of the equilateral triangle equals ……………

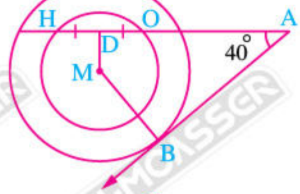
(a) 30° (b) 45° (c) 60° (d) 120°

6 If ABCD is a cyclic quadrilateral , m (∠ A) = 70° , then m (∠ C) = ……………

(a) 70° (b) 90° (c) 110° (d) 130°

---

2 [a] **In the opposite figure :**



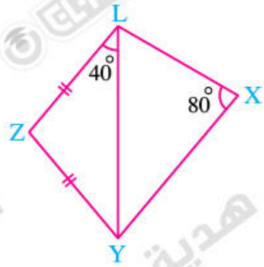
Two concentric circles at M

, D is the midpoint of $\overline{OH}$, m (∠ A) = 40°

, $\overrightarrow{AB}$ is a tangent to the big circle at B

Find with proof : m (∠ DMB)

[b] **In the opposite figure :**



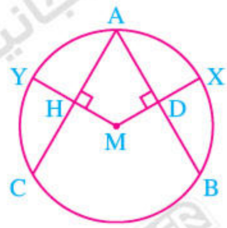
XYZL is a quadrilateral , m (∠ YLZ) = 40°

, m (∠ X) = 80° , LZ = ZY

Prove that : XYZL is a cyclic quadrilateral.

---

3 [a] **In the opposite figure :**

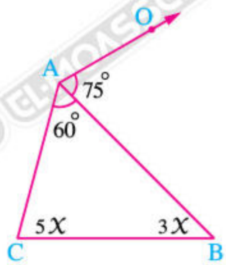


$\overline{AB}$, $\overline{AC}$ are two chords in the circle M

, AB = AC , $\overline{MD} \perp \overline{AB}$, $\overline{MH} \perp \overline{AC}$

Prove that : DX = HY

[b] **In the opposite figure :**



ABC is a triangle , m (∠ BAC) = 60° , m (∠ B) = 3 X°

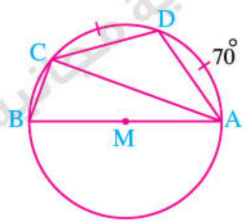
, m (∠ C) = 5 X° , m (∠ BAO) = 75°

Find with proof : the value of X

, then prove that : $\overrightarrow{AO}$ is a tangent to the circle which passes through the points A , B and C

---

4 [a] **In the opposite figure :**



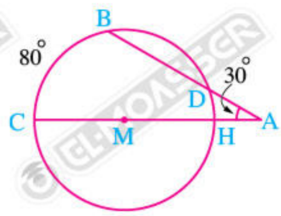
$\overline{AB}$ is a diameter in the circle M

, D is the midpoint of $\widehat{ADC}$

, m ($\widehat{AD}$) = 70°

Find with proof : m (∠ ADC) , m (∠ DCB)

[b] **In the opposite figure :**

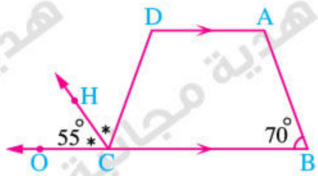


$\overline{AB} \cap \overline{AC} = \{A\}$, m (∠ A) = 30°

, m ($\widehat{BC}$) = 80°

Find with proof : m ($\widehat{HD}$)

5 [a] **In the opposite figure :**

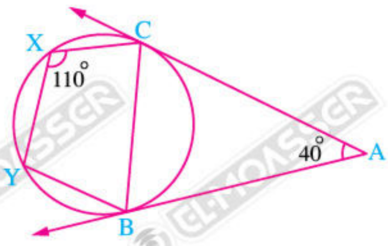


$\overrightarrow{AD} \,//\, \overrightarrow{BC}$, $\overrightarrow{CH}$ bisects ∠ DCO

, m (∠ B) = 70° , m (∠ OCH) = 55°

Prove that : ABCD is a cyclic quadrilateral.

[b] **In the opposite figure :**



$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C

, m (∠ A) = 40° , m (∠ X) = 110°

Prove that : $\overrightarrow{BC}$ bisects ∠ ABY

4 Beni Suef Governorate

Answer the following questions : *(Calculator is permitted)*

1 **Choose the correct answer from those given :**

1 ABCD is a square , then m (∠ ACB) = ...

(a) 90° (b) 60° (c) 45° (d) 30°

2 If the length of $\overline{AB}$ = 6 cm. , then the radius length of the smallest circle which passes through the two points A and B equals .... cm.

(a) 2 (b) 3 (c) 6 (d) 12

3 Δ ABC is right-angled at B , D is the midpoint of $\overline{AC}$, then BD = ...

(a) AB (b) BC (c) AC (d) AD

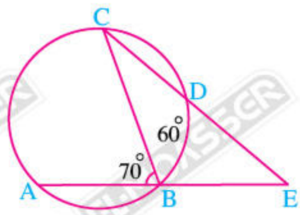
4 ABCD is a cyclic quadrilateral , if m (∠ ACB) = 80° , then m (∠ ADB) = ...

(a) 40° (b) 80° (c) 100° (d) 160°

5 If the ratio between the lengths of two corresponding sides of two similar polygons equals 3 : 4 , then the ratio between their perimeters equals ....

(a) 3 : 4 (b) 4 : 3 (c) 9 : 16 (d) 16 : 9

6 **In the opposite figure :**



$\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$, m (∠ ABC) = 70°

and m ($\overset{\frown}{DB}$) = 60° , then m (∠ E) = ...

(a) 40° (b) 60°

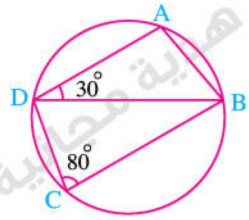
(c) 80° (d) 100°

2 [a] **In the opposite figure :**

ABCD is a quadrilateral inscribed in a circle

, m (∠ C) = 80° and m (∠ ADB) = 30°

Find : m (∠ ABD)

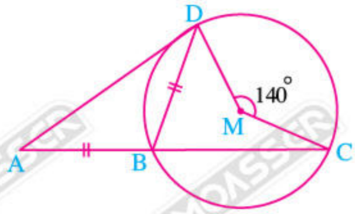


[b] **In the opposite figure :**

A circle with centre M , $\overrightarrow{CB} \cap \overrightarrow{DA} = \{A\}$

, m (∠ DMC) = 140° and AB = BD

Find : m (∠ BAD)

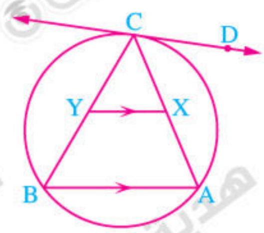


3 [a] **In the opposite figure :**

Δ ABC is inscribed in a circle

, $\overleftrightarrow{CD}$ is a tangent to the circle at C and $\overline{AB}$ // $\overline{XY}$

Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle passing through the vertices of Δ XYC



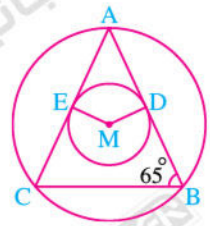
[b] **In the opposite figure :**

Two concentric circles M , m (∠ B) = 65°

, $\overline{AB}$ and $\overline{AC}$ are two tangent-segments

to the smaller circle at D and E

1 **Prove that :** AB = AC   2 **Find :** m (∠ BAC)



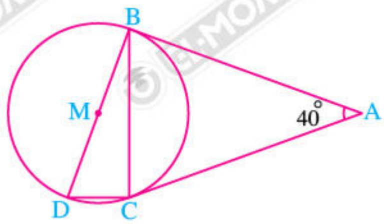
4 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments

to the circle M at B and C

, $\overline{BD}$ is a diameter in the circle and m (∠ BAC) = 40°

Find : m (∠ CBD)

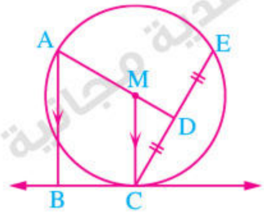


[b] **In the opposite figure :**

$\overleftrightarrow{BC}$ is a tangent to the circle M at C

, M ∈ $\overline{AD}$, D is the midpoint of $\overline{CE}$ and $\overline{AB}$ // $\overline{MC}$

Prove that : the figure ABCD is a cyclic quadrilateral.

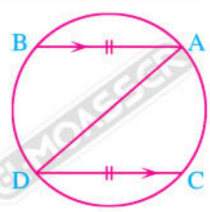


5 [a] **In the opposite figure :**

$\overline{AB}$ and $\overline{CD}$ are two chords in a circle

, AB = CD and $\overline{AB}$ // $\overline{CD}$

Prove that : $\overline{AD}$ is a diameter in the circle.



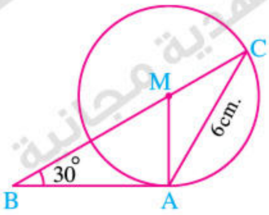
[b] **In the opposite figure :**

$\overleftrightarrow{AB}$ is a tangent to the circle M at A

, $\overrightarrow{AB} \cap \overrightarrow{CM} = \{B\}$, m (∠ ABC) = 30° and AC = 6 cm.

Find : 1 m (∠ AMC)

2 The length of $\overline{AB}$



5 Souhag Governorate

Answer the following questions : *(Calculator is permitted)*

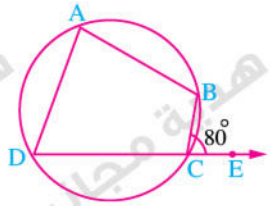
1 Choose the correct answer from the given ones :

1 **In the opposite figure :**

m (∠ A) = ...

(a) 100° (b) 180°

(c) 80° (d) 360°



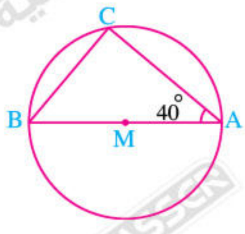
2 **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, m (∠ CAB) = 40° , then m (∠ B) = ...

(a) 50° (b) 40°

(c) 100° (d) 70°



3 The number of circles whose centre is (7 , 4) and pass through the point (3 , 1) is ....

(a) 2 (b) 1 (c) 5 (d) infinite.

4 The number of symmetry axes of an equilateral triangle equals ....

(a) 1 (b) 2 (c) 3 (d) 4

5 A circle is of circumference 6 π cm. , then the length of its diameter equals .... cm.

(a) 6 (b) 3 (c) 12 (d) 0

6 The area of the rhombus with diagonal lengths 6 cm. , 8 cm. is .... cm².

(a) 2 (b) 14 (c) 24 (d) 48

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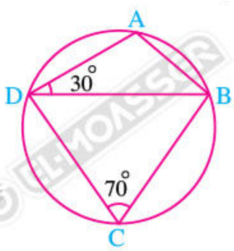
2 [a] **In the opposite figure :**

m (∠ ADB) = 30°

, m (∠ C) = 70°

Find : m (∠ ABD)

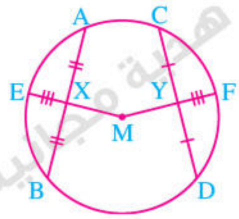


[b] **In the opposite figure :**

$\overline{AB}$, $\overline{CD}$ are two chords in the circle M , EX = YF

, X and Y are the midpoints of $\overline{AB}$ and $\overline{CD}$

Prove that : AB = CD



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3 [a] **In the opposite figure :**

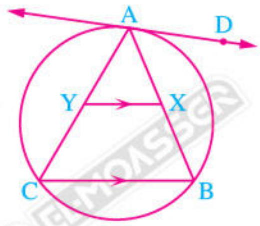
Δ ABC is inscribed in the circle

, $\overleftrightarrow{AD}$ is a tangent to the circle at A

, X $\in \overline{AB}$, Y $\in \overline{AC}$ where $\overline{XY} \mathbin{/\!/} \overline{BC}$

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y

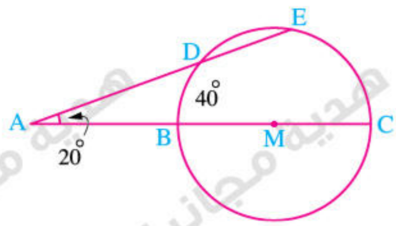


[b] **In the opposite figure :**

$\overline{AE} \cap \overline{AC} = \{A\}$, $\overline{BC}$ is a diameter in the circle M

, m ($\angle$ A) = 20° , m ($\overset{\frown}{BD}$) = 40°

Find : m ($\overset{\frown}{ED}$)



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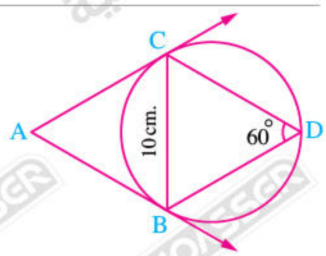
4 [a] **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle

, m ($\angle$ D) = 60° , CB = 10 cm.

Find : the perimeter of Δ ABC



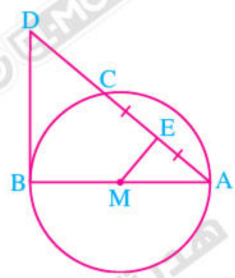
[b] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $\overline{BD}$ is a tangent-segment to the circle at B

, E is the midpoint of $\overline{AC}$

Prove that : the figure MEDB is a cyclic quadrilateral.



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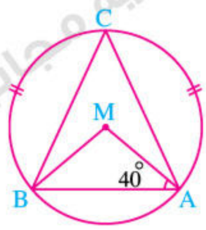
5 [a] **In the opposite figure :**

In circle M

, m ($\angle$ MAB) = 40°

, m ($\overset{\frown}{AC}$) = m ($\overset{\frown}{BC}$)

Find : m ($\angle$ CAM)



[b] M , N are two circles , their radii lengths are 8 cm. , 6 cm. respectively. **Determine the position of each of them with respect to the other in each of the following cases :**

1 MN = 2 cm. 2 MN = 6 cm. 3 MN $\notin$]0 , ∞[

6 Aswan Governorate

Answer the following questions :

1 Choose the correct answer from those given :

1 If 3 , 7 , X are side lengths of an isosceles triangle , then X = …………… cm.

(a) 3 (b) 4 (c) 7 (d) 10

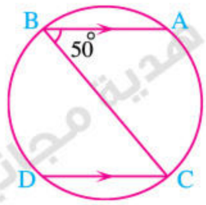
2 The square whose perimeter is 20 cm. , then its area is …………… cm².

(a) 4 (b) 5 (c) 20 (d) 25

3 The parallelogram in which one of its angles is right is ……………

(a) a rectangle. (b) a rhombus. (c) a square. (d) a trapezium.

4 **In the opposite figure :**



If $\overline{BA} \parallel \overline{CD}$, m (∠ B) = 50°

, then m $(\overset{\frown}{AC})$ + m $(\overset{\frown}{BD})$ = ……………

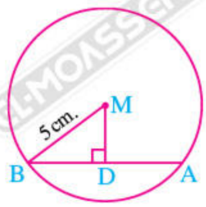
(a) 50° (b) 200°

(c) 150° (d) 180°

5 If $\overline{AC}$ is a diameter in a circle M , then m $(\overset{\frown}{AC})$ = ……………

(a) 45° (b) 90° (c) 180° (d) 360°

6 **In the opposite figure :**



If AB = 8 cm. , MB = 5 cm. , $\overline{MD} \perp \overline{AB}$

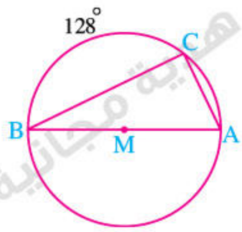
, then MD = …………… cm.

(a) 3 (b) 4

(c) 8 (d) 13

---

2 [a] In the opposite figure :

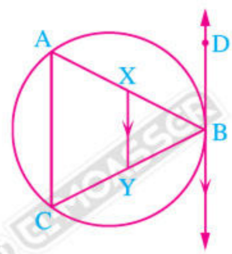


$\overline{AB}$ is a diameter in the circle M

, m $(\overset{\frown}{BC})$ = 128°

Find : m (∠ B)

[b] In the opposite figure :



ABC is an inscribed triangle in the circle

, $\overleftrightarrow{BD}$ is a tangent to the circle at B

, X ∈ $\overline{AB}$, Y ∈ $\overline{BC}$

where $\overline{XY} \parallel \overleftrightarrow{BD}$

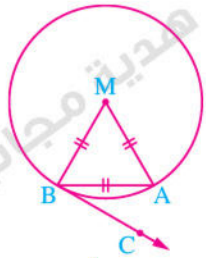
Prove that : the figure AXYC is a cyclic quadrilateral.

3 **[a] In the opposite figure :**

MAB is an equilateral triangle

, $\overrightarrow{BC}$ is a tangent to the circle at B

Find : m (∠ ABC)



[b] In the opposite figure :

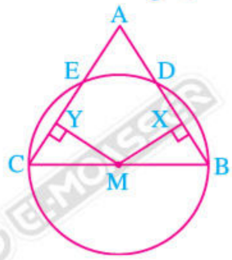
ABC is a triangle in which AB = AC

, a circle M was drawn whose diameter is $\overline{BC}$

and intersected $\overline{AB}$ at D , $\overline{AC}$ at E

, $\overline{MX} \perp \overline{BD}$, $\overline{MY} \perp \overline{CE}$

Prove that : BD = CE



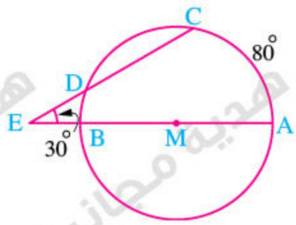
4 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, $\overrightarrow{AB} \cap \overrightarrow{CD} = \{E\}$

, m (∠ AEC) = 30° , m $(\overset{\frown}{AC})$ = 80°

Find : m $(\overset{\frown}{CD})$

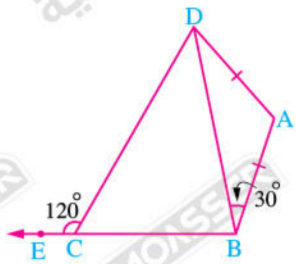


[b] In the opposite figure :

AB = AD , m (∠ ABD) = 30°

, m (∠ DCE) = 120°

Prove that : the figure ABCD is a cyclic quadrilateral.

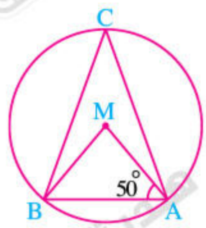


5 **[a] In the opposite figure :**

ABC is a triangle inscribed in a circle M

, m (∠ MAB) = 50°

Find : m (∠ C)



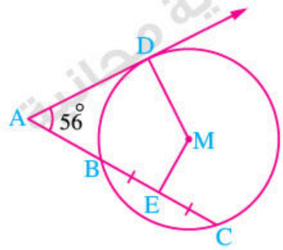
[b] In the opposite figure :

$\overrightarrow{AD}$ is a tangent to the circle M

, $\overrightarrow{AC}$ intersects the circle at B and C

, E is the midpoint of $\overline{BC}$, m (∠ A) = 56°

Find : m (∠ DME)



7 New Valley Governorate

Answer the following questions : *(Calculator is allowed)*

1 Choose the correct answer from those given :

1. The inscribed angle drawn in a semicircle is ……………

(a) acute. (b) obtuse. (c) straight. (d) right.

2. ABCD is a cyclic quadrilateral , m (∠ A) = 60° , then m (∠ C) = ……………

(a) 120° (b) 90° (c) 60° (d) 30°

3. The angle of tangency is the included angle between ……………

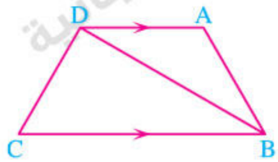
(a) two chords. (b) two tangents.

(c) a chord and a tangent. (d) a chord and a diameter.

4. M and N are two intersecting circles of radii lengths 3 cm. and 5 cm. , then MN ∈ ……………

(a)]8 , ∞[(b)]– ∞ , 2[(c)]0 , 2[(d)]2 , 8[

5. **In the opposite figure :**



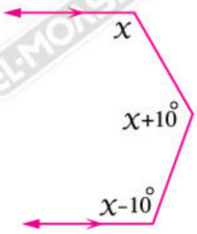
$AD = \frac{1}{2}$ CB

, then the area of Δ ABD : the area of the figure ABCD = ……………

(a) 1 : 2 (b) 1 : 3

(c) 1 : 4 (d) 2 : 3

6. **In the opposite figure :**



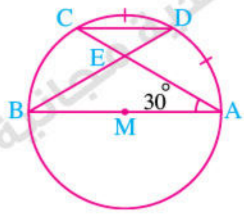
$\tan\left(\frac{x}{2}\right) =$ ……………

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$

(c) $\sqrt{3}$ (d) 1

---

2 [a] In the opposite figure :



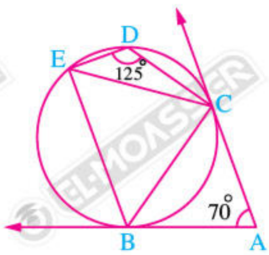
$\overline{AB}$ is a diameter in the circle M , C ∈ the circle

, m (∠ CAB) = 30° , D is the midpoint of $\widehat{AC}$

, $\overline{DB} \cap \overline{AC} = \{E\}$

1. **Find :** m $(\widehat{AD})$
2. **Prove that :** $\overline{AB} \,//\, \overline{DC}$

[b] In the opposite figure :



$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C respectively

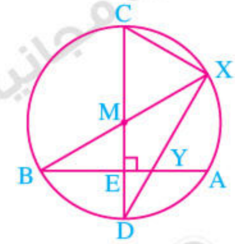
, m (∠ A) = 70° , m (∠ CDE) = 125°

Prove that : 1. CB = CE 2. $\overrightarrow{AC} \,//\, \overline{BE}$

3 [a] State two cases of the cyclic quadrilateral.

[b] **In the opposite figure :**

$\overline{AB}$ is a chord in the circle M , $\overline{CD}$ is a diameter perpendicular to $\overline{AB}$ and intersects it at E , $\overrightarrow{BM}$ intersects the circle at X , $\overline{XD} \cap \overline{AB} = \{Y\}$



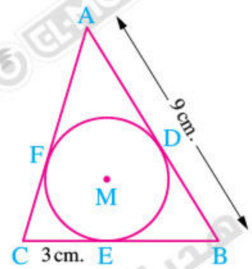
Prove that : 1 The figure XYEC is a cyclic quadrilateral.

2 m (∠ DYB) = m (∠ DBX)

---

4 [a] **In the opposite figure :**

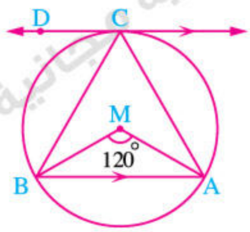
A circle is drawn touching the sides of the triangle ABC $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ at D , E , F respectively , AB = 9 cm. , CE = 3 cm.



Find : the perimeter of Δ ABC

[b] **In the opposite figure :**

$\overleftrightarrow{CD}$ is a tangent to the circle at C , $\overleftrightarrow{CD}$ // $\overline{AB}$, m (∠ AMB) = 120°

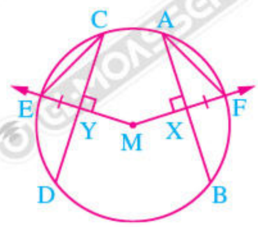


Prove that : Δ ABC is an equilateral triangle.

---

5 [a] **In the opposite figure :**

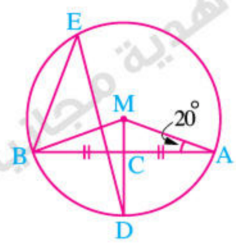
$\overline{AB}$, $\overline{CD}$ are two chords in the circle M , $\overrightarrow{MX} \perp \overline{AB}$ and intersects the circle at F , $\overrightarrow{MY} \perp \overline{CD}$ and intersects the circle at E , FX = EY



Prove that : 1 AB = CD   2 AF = CE

[b] **In the opposite figure :**

C is the midpoint of $\overline{AB}$, $\overrightarrow{MC} \cap$ the circle M = {D} , m (∠ MAB) = 20°



Find : m (∠ BED) , m ($\overset{\frown}{ADB}$)

8 South Sinai Governorate

Answer the following questions :

1 Choose the correct answer from those given :

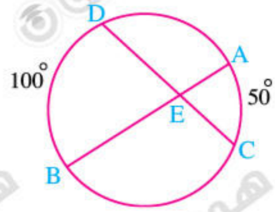
1. The measure of the arc which equals half the measure of the circle equals ……………

(a) 360° (b) 180° (c) 120° (d) 90°

2. The number of common tangents of two circles touching internally equals ……………

(a) zero (b) 1 (c) 2 (d) 3

3. **In the opposite figure :**



m (∠ AEC) = ……………

(a) 25° (b) 50°

(c) 75° (d) 100°

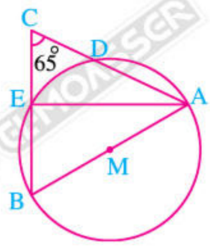
4. If ABCD is a cyclic quadrilateral in which m (∠ B) = 50° , then m (∠ D) = ……………

(a) 25° (b) 50° (c) 100° (d) 130°

5. If the lengths of the two diagonals of a rhombus are 3 cm. and 4 cm.
, then its area = …………… cm^2

(a) 6 (b) 12 (c) 24 (d) 48

6. **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M

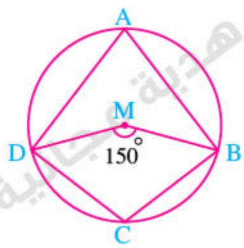
, m (∠ C) = 65° , then m (∠ CAE) = ……………

(a) 65° (b) 45°

(c) 30° (d) 25°

---

2 [a] In the opposite figure :

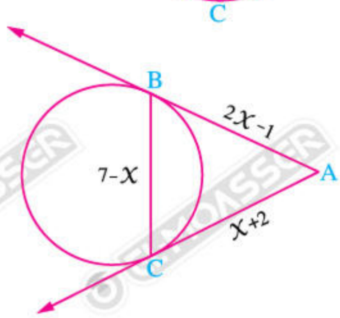


m (∠ BMD) = 150°

Find : 1 m (∠ BAD)

2 m (∠ BCD)

[b] In the opposite figure :



$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle

where AB = (2 X – 1) cm.

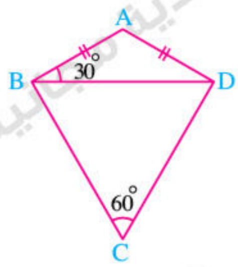
, AC = (X + 2) cm. , BC = (7 – X) cm.

Find : 1 The value of X

2 The perimeter of Δ ABC

3 [a] State two cases of the quadrilateral to be cyclic.

[b] **In the opposite figure :**



ABCD is a quadrilateral in which

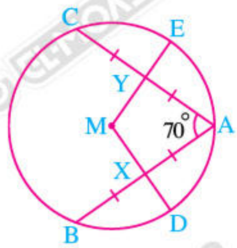
AB = AD , m (∠ ABD) = 30°

, m (∠ C) = 60°

Prove that : the figure ABCD is a cyclic quadrilateral.

---

4 [a] **In the opposite figure :**



$\overline{AB}$, $\overline{AC}$ are two chords equal in length in the circle M

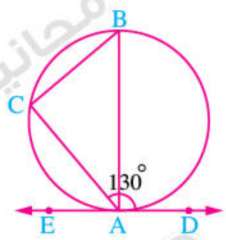
, X is the midpoint of $\overline{AB}$

, Y is the midpoint of $\overline{AC}$, m (∠ A) = 70°

1 **Calculate :** m (∠ DME)

2 **Prove that :** XD = YE

[b] **In the opposite figure :**



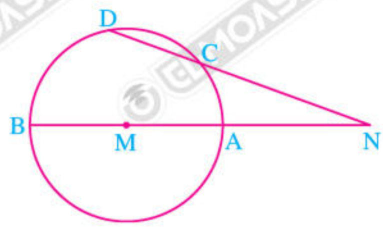
$\overleftrightarrow{AD}$ is a tangent to the circle at A

, m (∠ CAD) = 130°

Find with proof : m (∠ ABC)

---

5 [a] **In the opposite figure :**

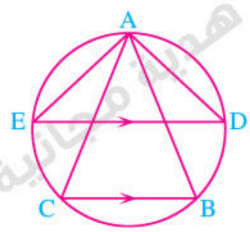


$\overline{AB}$ is a diameter in the circle M

, $\overrightarrow{BA} \cap \overrightarrow{DC} = \{N\}$

Prove that : NC > NA

[b] **In the opposite figure :**



ABC is a triangle inscribed in the circle

, $\overline{DE}$ // $\overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)

9 North Sinai Governorate

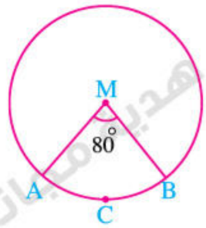
Answer the following questions :

1 Choose the correct answer from those given :

1. If the surface of circle M $\cap$ the surface of circle N = $\{A\}$, then the two circles are ....
   (a) distant. (b) touching externally.
   (c) touching internally. (d) intersecting.

2. The sum of measures of the interior angles of the triangle equals ....
   (a) 90° (b) 180° (c) 360° (d) 60°

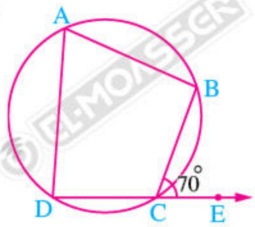
3. **In the opposite figure :**

   In the circle M , m ($\angle$ AMB) = 80°
   , then m $(\overset{\frown}{ACB})$ = ...
   (a) 40° (b) 160°
   (c) 80° (d) 120°

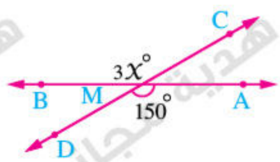
4. The area of the square whose side length is 5 cm. equals .... cm^2.
   (a) 25 (b) 5 (c) 20 (d) 10

5. **In the opposite figure :**

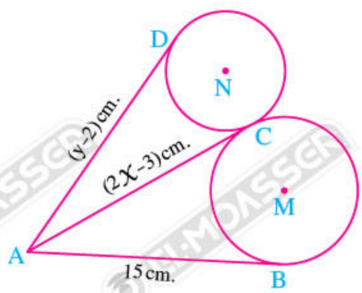
   E $\in \overrightarrow{DC}$, m ($\angle$ BCE) = 70°
   , then m ($\angle$ BAD) = ...
   (a) 70° (b) 140°
   (c) 35° (d) 120°

6. **In the opposite figure :**

   $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{M\}$, then X = ...
   (a) 150° (b) 120°
   (c) 50° (d) 360°

2 [a] In the opposite figure :



M , N are two circles touching externally at C
, $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M
, $\overline{AC}$, $\overline{AD}$ are two tangent-segments to the circle N
, AB = 15 cm. , AC = (2 X – 3) cm. , AD = (y – 2) cm.

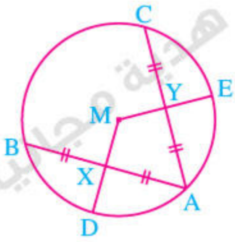
Find with proof : the value of each of X , y

[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two equal chords in the circle M

, X , Y are the midpoints of $\overline{AB}$, $\overline{AC}$ respectively

Prove that : XD = YE



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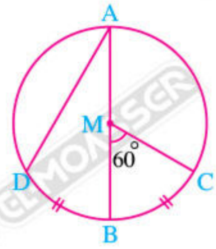
3 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

, m $(\widehat{BC})$ = m $(\widehat{BD})$

, m (∠ BMC) = 60°

Find with proof : m (∠ BAD)

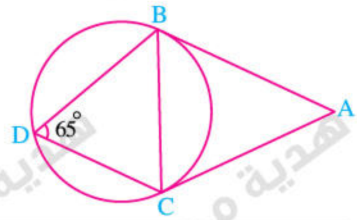


[b] In the opposite figure :

$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle at B , C

, m (∠ BDC) = 65°

Find with proof : m (∠ BAC)



---

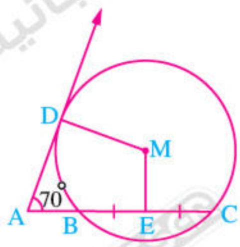
4 **[a] In the opposite figure :**

$\overrightarrow{AD}$ is a tangent to the circle at D

, $\overrightarrow{AC}$ is a secant of the circle at B , C

, E is the midpoint of $\overline{BC}$, m (∠ DAC) = 70°

Find with proof : m (∠ DME)



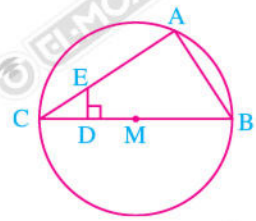
[b] In the opposite figure :

$\overline{BC}$ is a diameter in the circle M

, $\overline{ED} \perp \overline{BC}$

Prove that : 1 ABDE is a cyclic quadrilateral.

2 m (∠ CED) = $\frac{1}{2}$ m $(\widehat{AC})$



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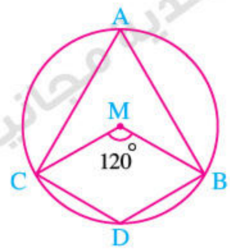
5 **[a] In the opposite figure :**

M is a circle in which m (∠ BMC) = 120°

Find with proof :

1 m (∠ BAC)

2 m (∠ BDC)

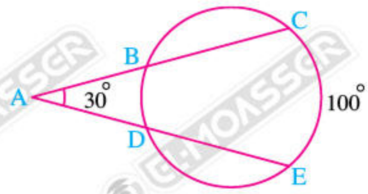


[b] In the opposite figure :

m $(\widehat{CE})$ = 100°

, m (∠ A) = 30°

Find with proof : m $(\widehat{BD})$



10 Red Sea Governorate

Answer the following questions :

1 Choose the correct answer from the given answers :

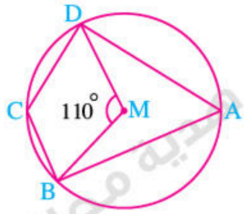
1. Two circles M and N touching internally , their radii lengths are 5 cm. and 9 cm. respectively , then MN = ... cm.

(a) 14 (b) 4 (c) 5 (d) 9

2. ABC is a triangle in which m (∠ A) = 50° and m (∠ B) = 70° , then the number of axes of symmetry of Δ ABC is ....

(a) zero (b) 1 (c) 2 (d) 3

3. **In the opposite figure :**



m (∠ BMD) = 110°

, then m (∠ C) = ...

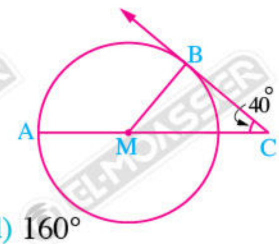
(a) 50° (b) 70°

(c) 110° (d) 125°

4. A rhombus of side length L cm. , its perimeter = ... cm.

(a) L^2 (b) $2\,L^2$ (c) $4\,L$ (d) $2\sqrt{2}\,L$

5. **In the opposite figure :**



$\overrightarrow{CB}$ is a tangent to the circle M

, m (∠ C) = 40° , M ∈ $\overline{AC}$

, then m $(\overset{\frown}{AB})$ = ...

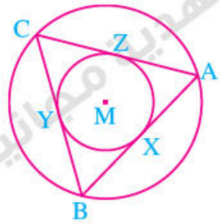
(a) 40° (b) 80° (c) 130° (d) 160°

6. If ∠ A and ∠ B are two supplementary angles , then m (∠ A) + m (∠ B) = ...

(a) 90° (b) 180° (c) 270° (d) 360°

---

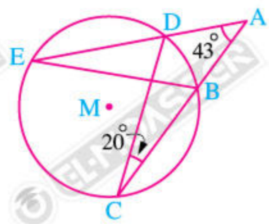
2 [a] In the opposite figure :



Two concentric circles M , ABC is a triangle its vertices are on the greater circle and its sides touch the smaller circle at X , Y , Z

Prove that : ABC is an equilateral triangle.

[b] In the opposite figure :



$\overrightarrow{ED} \cap \overrightarrow{CB} = \{A\}$

, m (∠ A) = 43°

and m (∠ ACD) = 20°

Find : m (∠ E) and m $(\overset{\frown}{CE})$

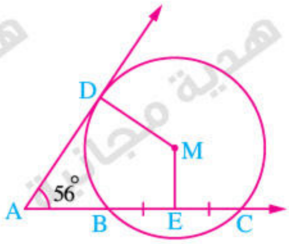
3 [a] **In the opposite figure :**

$\overrightarrow{AD}$ is a tangent to the circle M at D

, m (∠ A) = 56°

, E is the midpoint of $\overline{CB}$

Find : m (∠ M)

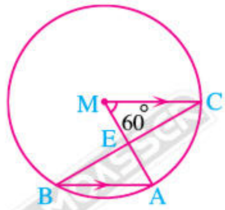


[b] **In the opposite figure :**

$\overline{MC}$ // $\overline{BA}$, $\overline{BC} \cap \overline{MA} = \{E\}$

, m (∠ M) = 60°

Find : m (∠ B) **and prove that :** $\overline{BC} \perp \overline{AM}$



4 [a] **In the opposite figure :**

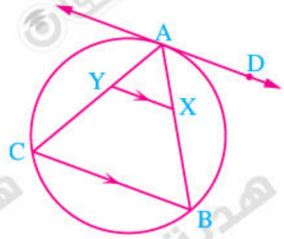
$\overleftrightarrow{AD}$ is a tangent to the circle at A

, X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$

, where $\overline{XY}$ // $\overline{BC}$

Prove that :

$\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y



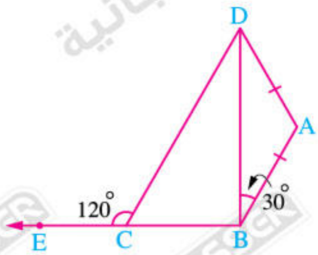
[b] **In the opposite figure :**

ABCD is a quadrilateral in which E ∈ $\overrightarrow{BC}$

, AB = AD , m (∠ DCE) = 120°

, m (∠ ABD) = 30°

Prove that : ABCD is a cyclic quadrilateral.



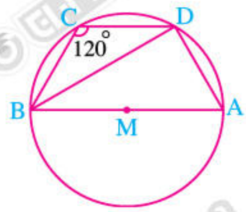
5 [a] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M

and m (∠ BCD) = 120°

Find : 1 m (∠ A)

2 m (∠ ABD)

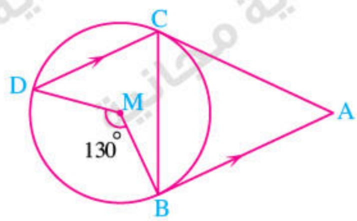


[b] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M at B and C

, $\overline{AB}$ // $\overline{CD}$, m (∠ BMD) = 130°

Find : m (∠ A)



1 Suez

1

1 b | 2 c | 3 a
4 b | 5 b | 6 d

2

[a] In the greater circle :

$\because \overline{ME} \perp \overline{AB}$ $\quad \therefore$ E is the midpoint of $\overline{AB}$

$\therefore AE = BE$ (1)

In the smaller circle :

$\because \overline{ME} \perp \overline{CD}$ $\quad \therefore$ E is the midpoint of $\overline{CD}$

$\therefore CE = DE$ (2)

Subtracting (2) from (1) : $\therefore AC = BD$ (Q.E.D.)

[b] $\because \overline{AB}$ is a diameter

$\therefore m(\angle C) = 90^\circ$ (First req.)

$, \because m(\angle B) = \frac{1}{2}\, m(\overset{\frown}{AC}) = \frac{1}{2} \times 120^\circ = 60^\circ$

$\therefore$ In Δ ACB : $m(\angle A) = 180^\circ - (90^\circ + 60^\circ) = 30^\circ$

(Second req.)

3

[a] $\because \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

$, MX = MY$ $\quad \therefore AB = CD$

$, \because \overline{MY} \perp \overline{CD}$ $\quad \therefore$ Y is the midpoint of $\overline{CD}$

$\therefore AB = CD = 2 \times 7 = 14$ cm. (The req.)

[b] In Δ ABC :

$m(\angle C) = 180^\circ - (70^\circ + 60^\circ) = 50^\circ$

$, \because \overrightarrow{AD}$ is a tangent to the circle M at A

$\therefore m(\angle BAD)$ (tangency) $= m(\angle C)$ (inscribed) $= 50^\circ$

(The req.)

4

[a] State by yourself.

[b] $\because \overrightarrow{CF}$ bisects $\angle$ DCE

$\therefore m(\angle DCE) = 2 \times 50^\circ = 100^\circ$

$, \because \overline{AD} \,/\!/\, \overline{BC}$, $\overleftrightarrow{CD}$ is a transversal

$\therefore m(\angle D) = m(\angle DCE) = 100^\circ$ (alternate angles)

$\therefore m(\angle B) + m(\angle D) = 80^\circ + 100^\circ = 180^\circ$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

5

[a] $m(\angle AMB) = 2\, m(\angle ACB) = 2 \times 45^\circ = 90^\circ$

(central and inscribed angles subtended by $\overset{\frown}{AB}$)

(First req.)

$, \because MA = MB = r$

$\therefore$ In Δ AMB :

$m(\angle MAB) = m(\angle MBA) = \frac{180^\circ - 90^\circ}{2} = 45^\circ$

(Second req.)

[b] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents.

$\therefore AB = AC$

$\therefore$ In Δ ABC :

$m(\angle ABC) = m(\angle ACB) = \frac{180 - 40^\circ}{2} = 70^\circ$

$, \because$ CEDB is a cyclic quadrilateral

$\therefore m(\angle DBC) = 180^\circ - 110^\circ = 70^\circ$

$\therefore m(\angle ABC) = m(\angle DBC)$ (Q.E.D.)

2 Kafr El-Sheikh

1

1 b | 2 c | 3 d
4 b | 5 c | 6 a

2

[a] $\because XY = XZ$

$, \overrightarrow{MA} \perp \overline{XY}$, $\overrightarrow{MB} \perp \overline{XZ}$

$\therefore MA = MB$

$, \because MC = MD = r$

Subtracting : $\therefore AC = BD$ (Q.E.D.)

[b] $\because$ A is the midpoint of $\overline{XB}$

$\therefore \overline{MA} \perp \overline{XB}$ $\quad \therefore m(\angle MAB) = 90^\circ$

$, \because \overline{YZ}$ is a tangent-segment to the circle at Y

$\therefore \overline{MY} \perp \overline{YZ}$

$\therefore m(\angle MYZ) = 90^\circ$

$\therefore m(\angle MAB) + m(\angle MYZ) = 90^\circ + 90^\circ = 180^\circ$

$\therefore$ MAZY is a cyclic quadrilateral. (Q.E.D.)

3

[a] $\because \overline{AD}$, $\overline{AF}$ are two tangent-segments to the circle.

$\therefore AD = AF = 3$ cm.

$, \because \overline{BD}$, $\overline{BE}$ are two tangent-segments to the circle.

$\therefore BD = BE = 4$ cm.

, $\because \overline{CE}$, $\overline{CF}$ are two tangent-segments to the circle.

$\therefore CE = CF = 5$ cm.

$\therefore$ The perimeter of $\Delta ABC = 3 + 3 + 4 + 4 + 5 + 5$

$= 24$ cm. (The req.)

[b] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle C) = 180° - 100° = 80°$

$\therefore$ In Δ CBD :

$\because BC = DC$

$\therefore m(\angle CBD) = m(\angle CDB) = \frac{180° - 80°}{2}$

$= 50°$ (The req.)

4

[a] $\because m(\overset{\frown}{ABC}) = m(\angle CMA) = 100°$

$\therefore m(\overset{\frown}{AC}$ the major$) = 360° - 100° = 260°$

$\therefore m(\angle ABC) = \frac{1}{2}\, m(\overset{\frown}{AC}$ the major$)$

$= \frac{1}{2} \times 260° = 130°$ (The req.)

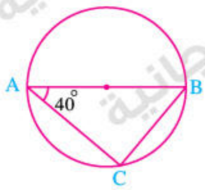
[b] $\because \overline{AB}$ is a diameter

$\therefore m(\angle ACB) = 90°$

$\therefore$ In Δ ACB :

$m(\angle B) = 180° - (90° + 40°)$

$= 50°$



(The req.)

5

[a] $\because \overrightarrow{BC}$ is a tangent to the circle at B

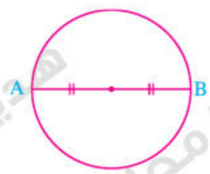
$\therefore m(\angle D)$ (inscribed) $= m(\angle ABC)$ (tangency) $= 70°$

(First req.)

$\therefore m(\overset{\frown}{AB}$ the minor$) = 2\, m(\angle D) = 2 \times 70°$

$= 140°$ (Second req.)

[b]



We can draw one circle.

3 El-Fayoum

1

1 b	2 c	3 b
4 c	5 d	6 c

2

[a] $\because$ D is the midpoint of $\overline{OH}$

$\therefore \overline{MD} \perp \overline{OH}$ $\therefore m(\angle MDA) = 90°$

, $\because \overrightarrow{AB}$ is a tangent to the big circle at B

$\therefore \overline{MB} \perp \overrightarrow{AB}$ $\therefore m(\angle MBA) = 90°$

$\therefore$ From the quadrilateral ABMD :

$m(\angle DMB) = 360° - (90° + 90° + 40°)$

$= 140°$ (The req.)

[b] In Δ ZLY : $\because LZ = ZY$

$\therefore m(\angle ZYL) = m(\angle ZLY) = 40°$

$\therefore m(\angle Z) = 180° - 2 \times 40° = 100°$

$\therefore m(\angle X) + m(\angle Z) = 80° + 100° = 180°$

$\therefore$ XYZL is a cyclic quadrilateral. (Q.E.D.)

3

[a] $\because AB = AC$

, $\overline{MD} \perp \overline{AB}$, $\overline{MH} \perp \overline{AC}$

$\therefore MD = MH$, $\because MX = MY = r$

Subtracting : $\therefore DX = HY$ (Q.E.D.)

[b] In Δ ABC :

$\because m(\angle CAB) + m(\angle C) + m(\angle B) = 180°$

$\therefore 60° + 5x° + 3x° = 180°$

$\therefore 8x = 180° - 60° = 120°$

$\therefore x = 15°$ (First req.)

$\therefore m(\angle C) = 5 \times 15° = 75°$

$\therefore m(\angle OAB) = m(\angle C)$

$\therefore \overrightarrow{AO}$ is a tangent to the circle which passes through the points A , B and C (Second req.)

4

[a] $\because m(\overset{\frown}{AD}) = m(\overset{\frown}{CD}) = 70°$

$\therefore m(\angle ACD) = m(\angle CAD) = \frac{1}{2} \times 70° = 35°$

$\therefore$ In Δ ACD :

$m(\angle ADC) = 180° - (35° + 35°) = 110°$ (First req.)

, $\because \overline{AB}$ is a diameter

$\therefore m(\angle ACB) = 90°$

$\therefore m(\angle DCB) = 90° + 35° = 125°$ (Second req.)

[b] $\because m(\angle A) = \frac{1}{2}\left[m(\overset{\frown}{BC}) - m(\overset{\frown}{HD})\right]$

$\therefore 30° = \frac{1}{2}\left[80° - m(\overset{\frown}{HD})\right]$

$\therefore 60° = 80° - m(\overset{\frown}{HD})$

$\therefore m(\overset{\frown}{HD}) = 80° - 60° = 20°$ (The req.)

5

[a] $\because \overrightarrow{CH}$ bisects $\angle$ DCO

$\therefore m(\angle DCO) = 2 \times 55° = 110°$

, $\because \overline{AD} \,/\!/\, \overline{BC}$, $\overleftrightarrow{DC}$ is a transversal

$\therefore m(\angle D) = m(\angle DCO) = 110^\circ$ (alternate angles)

$\therefore m(\angle B) + m(\angle D) = 70^\circ + 110^\circ = 180^\circ$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

[b] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents.

$\therefore AB = AC$

$\therefore$ In ΔABC :

$m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 40^\circ}{2} = 70^\circ$

, $\because$ BCXY is a cyclic quadrilateral

$\therefore m(\angle CBY) = 180^\circ - 110^\circ = 70^\circ$

$\therefore m(\angle ABC) = m(\angle CBY)$

$\therefore \overrightarrow{BC}$ bisects $\angle ABY$ (Q.E.D.)

4 Beni Suef

1

1 c 2 b 3 d

4 b 5 a 6 a

2

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle A) = 180^\circ - 80^\circ = 100^\circ$

$\therefore$ In ΔABD :

$m(\angle ABD) = 180^\circ - (30^\circ + 100^\circ) = 50^\circ$ (The req.)

[b] $\because m(\angle DBC) = \frac{1}{2} m(\angle DMC) = \frac{1}{2} \times 140^\circ = 70^\circ$

(inscribed and central angles subtended by $\overset{\frown}{CD}$)

$\therefore m(\angle DBA) = 180^\circ - 70^\circ = 110^\circ$

$\therefore$ In ΔABD : $\because AB = BD$

$\therefore m(\angle BAD) = m(\angle BDA) = \frac{180^\circ - 110^\circ}{2} = 35^\circ$

(The req.)

3

[a] $\because \overline{XY} // \overline{AB}$, $\overleftrightarrow{CB}$ is a transversal

$\therefore m(\angle CYX) = m(\angle B)$ (corresponding angles)

, $\because m(\angle B)$ (inscribed) $= m(\angle ACD)$ (tangency)

$\therefore m(\angle CYX) = m(\angle XCD)$

$\therefore \overleftrightarrow{CD}$ is a tangent to the circle passing through the vertices of ΔXYC (Q.E.D.)

[b] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the smaller circle at D , E

$\therefore \overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

, $\because MD = ME$ (radii lengths of the smaller circle)

$\therefore AB = AC$ (First req.)

$\therefore$ In ΔABC :

$m(\angle C) = m(\angle B) = 65^\circ$

$\therefore m(\angle BAC) = 180^\circ - 2 \times 65^\circ = 50^\circ$ (Second req.)

4

[a] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle.

$\therefore AB = AC$

$\therefore$ In ΔABC : $m(\angle ABC) = m(\angle ACB) = \frac{180^\circ - 40^\circ}{2}$

$= 70^\circ$

, $\because \overline{AB}$ is a tangent-segment at B

$\therefore \overline{MB} \perp \overline{AB}$ $\therefore m(\angle MBA) = 90^\circ$

$\therefore m(\angle CBD) = 90^\circ - 70^\circ = 20^\circ$ (The req.)

[b] $\because$ D is the midpoint of $\overline{CE}$

$\therefore \overline{MD} \perp \overline{CE}$ $\therefore m(\angle MDC) = 90^\circ$ (1)

, $\because \overleftrightarrow{BC}$ is a tangent to the circle at C

$\therefore \overline{MC} \perp \overleftrightarrow{BC}$

$\therefore m(\angle MCB) = 90^\circ$

, $\because \overline{AB} // \overline{MC}$, $\overleftrightarrow{BC}$ is a transversal

$\therefore m(\angle CBA) + m(\angle MCB) = 180^\circ$

(two interior angles in the same side of the transversal)

$\therefore m(\angle CBA) = 180^\circ - 90^\circ = 90^\circ$ (2)

From (1) and (2) :

$\therefore m(\angle MDC) + m(\angle CBA) = 90^\circ + 90^\circ = 180^\circ$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

5

[a] $\because AB = CD$

$\therefore m(\overset{\frown}{AB}) = m(\overset{\frown}{CD})$ (1)

, $\because \overline{AB} // \overline{CD}$

$\therefore m(\overset{\frown}{BD}) = m(\overset{\frown}{AC})$ (2)

Adding (1) and (2) :

$\therefore m(\overset{\frown}{ABD}) = m(\overset{\frown}{ACD})$

$\therefore \overline{AD}$ is a diameter in the circle. (Q.E.D.)

[b] $\because \overleftrightarrow{AB}$ is a tangent to the circle at A

$\therefore \overline{MA} \perp \overleftrightarrow{AB}$

$\therefore m(\angle MAB) = 90^\circ$

, $\because \angle AMC$ is exterior of ΔAMB

$\therefore m(\angle AMC) = m(\angle MAB) + m(\angle B)$

$= 90^\circ + 30^\circ = 120^\circ$ (First req.)

$\therefore$ In ΔAMC :

$\because MC = MA = r$

$\therefore m(\angle MCA) = m(\angle MAC) = \frac{180^\circ - 120^\circ}{2} = 30^\circ$

$\therefore$ In ΔABC : $m(\angle B) = m(\angle ACB)$

$\therefore AB = AC = 6$ cm. (Second req.)

5 Souhag

1

1 c 2 a 3 b

4 c 5 a 6 c

2

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle A) = 180° - 70° = 110°$

$\therefore$ In ΔABD :

$m(\angle ABD) = 180° - (30° + 110°) = 40°$ (The req.)

[b] $\because$ X is the midpoint of $\overline{AB}$ $\therefore \overline{MX} \perp \overline{AB}$

, $\because$ Y is the midpoint of $\overline{CD}$ $\therefore \overline{MY} \perp \overline{CD}$

, $\because XE = YF$, $ME = MF = r$

Subtracting : $\therefore MX = MY$

$\therefore AB = CD$ (Q.E.D.)

3

[a] $\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle AYX) = m(\angle C)$ (corresponding angles)

, $\because m(\angle C)$ (inscribed) $= m(\angle BAD)$ (tangency)

$\therefore m(\angle AYX) = m(\angle XAD)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] $\because m(\angle A) = \frac{1}{2}[m(\widehat{EC}) - m(\widehat{DB})]$

$\therefore 20° = \frac{1}{2}[m(\widehat{EC}) - 40°]$

$\therefore 40° = m(\widehat{EC}) - 40°$

$\therefore m(\widehat{EC}) = 40° + 40° = 80°$

, $\because \overline{BC}$ is a diameter

$\therefore m(\widehat{BC}) = 180°$

$\therefore m(\widehat{ED}) = 180° - (80° + 40°) = 60°$ (The req.)

4

[a] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle.

$\therefore AB = AC$

$\therefore \Delta ABC$ is an isosceles triangle

, $\because m(\angle CBA)$ (tangency) $= m(\angle D)$ (inscribed) $= 60°$

$\therefore \Delta ABC$ is an equilateral triangle.

$\therefore$ The perimeter of $\Delta ABC = 3 \times 10 = 30$ cm. (The req.)

[b] $\because$ E is the midpoint of $\overline{AC}$

$\therefore \overline{ME} \perp \overline{AC}$ $\therefore m(\angle MED) = 90°$

, $\because \overline{BD}$ is a tangent-segment to the circle M at B

$\therefore \overline{MB} \perp \overline{BD}$ $\therefore m(\angle MBD) = 90°$

$\therefore m(\angle MED) + m(\angle MBD) = 90° + 90° = 180°$

$\therefore$ MEDB is a cyclic quadrilateral. (Q.E.D.)

5

[a] In ΔABM :

$\because MA = MB = r$

$\therefore m(\angle MAB) = m(\angle MBA) = 40°$

$\therefore m(\angle AMB) = 180° - 2 \times 40° = 100°$

$\therefore m(\angle ACB) = \frac{1}{2} m(\angle AMB) = \frac{1}{2} \times 100° = 50°$

(inscribed and central angles subtended by $\widehat{AB}$)

, $\because m(\widehat{AC}) = m(\widehat{BC})$ $\therefore AC = BC$

$\therefore$ In ΔABC :

$m(\angle BAC) = m(\angle ABC) = \frac{180° - 50°}{2} = 65°$

$\therefore m(\angle CAM) = 65° - 40° = 25°$ (The req.)

[b] 1 $\because r_1 - r_2 = 8 - 6 = 2$ cm.

$\therefore MN = r_1 - r_2$

$\therefore$ The two circles are touching internally.

2 $\because r_1 + r_2 = 8 + 6 = 14$ cm. , $r_1 - r_2 = 2$ cm.

$\therefore r_1 - r_2 < MN < r_1 + r_2$

$\therefore$ The two circles are intersecting.

3 $MN \notin]0 , \infty[$

$\therefore$ The two circles are concentric.

6 Aswan

1

1 c	2 d	3 a
4 b	5 c	6 a

2

[a] $\because \overline{AB}$ is a diameter

$\therefore m(\widehat{AC}) = 180° - 128° = 52°$

$\therefore m(\angle B) = \frac{1}{2} m(\widehat{AC}) = \frac{1}{2} \times 52° = 26°$ (The req.)

[b] $\because \overline{XY} \parallel \overleftrightarrow{BD}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore m(\angle DBX) = m(\angle YXB)$ (alternate angles) (1)

, $\because m(\angle C)$ (inscribed) $= m(\angle ABD)$ (tangency) (2)

From (1) and (2) :

$\therefore m(\angle C) = m(\angle YXB)$

$\therefore$ AXYC is a cyclic quadrilateral. (Q.E.D.)

3

[a] $\because \Delta MAB$ is an equilateral triangle.

$\therefore m(\angle AMB) = 60°$

, $\because \overrightarrow{BC}$ is a tangent to the circle at B

$\therefore m(\angle ABC)$ (tangency) $= \frac{1}{2} m(\angle AMB)$ (central)

$= \frac{1}{2} \times 60° = 30°$ (The req.)

[b] In ΔABC : $\because AB = AC$

$\therefore m(\angle B) = m(\angle C)$

$\therefore m(\widehat{EB}) = m(\widehat{CD})$

By subtracting $m(\widehat{ED})$ from both sides

$\therefore m(\widehat{DB}) = m(\widehat{CE})$

$\therefore BD = CE$ (Q.E.D.)

4

[a] $\because m(\angle E) = \frac{1}{2}\left[m(\overset{\frown}{AC}) - m(\overset{\frown}{BD})\right]$

$\therefore 30^\circ = \frac{1}{2}\left[80^\circ - m(\overset{\frown}{BD})\right]$

$\therefore 60^\circ = 80^\circ - m(\overset{\frown}{BD})$

$\therefore m(\overset{\frown}{BD}) = 80^\circ - 60^\circ = 20^\circ$

, $\because \overline{AB}$ is a diameter $\therefore m(\overset{\frown}{AB}) = 180^\circ$

$\therefore m(\overset{\frown}{CD}) = 180^\circ - (80^\circ + 20^\circ) = 80^\circ$ (The req.)

[b] In ΔABD : $\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB) = 30^\circ$

$\therefore m(\angle A) = 180^\circ - 2 \times 30^\circ = 120^\circ$

$\therefore m(\angle A) = m(\angle DCE)$

$\therefore$ ABCD is a cyclic quadrilateral (Q.E.D.)

5

[a] In ΔABM : $\because MA = MB = r$

$\therefore m(\angle MAB) = m(\angle MBA) = 50^\circ$

$\therefore m(\angle M) = 180^\circ - 2 \times 50^\circ = 80^\circ$

$\therefore m(\angle C) = \frac{1}{2}\, m(\angle M) = \frac{1}{2} \times 80^\circ = 40^\circ$

(inscribed and central angles subtended by $\overset{\frown}{AB}$)

(The req.)

[b] $\because$ E is the midpoint of $\overline{BC}$

$\therefore \overline{ME} \perp \overline{BC}$ $\therefore m(\angle MEA) = 90^\circ$

, $\because \overrightarrow{AD}$ is a tangent to the circle at D

$\therefore \overline{MD} \perp \overrightarrow{AD}$

$\therefore m(\angle MDA) = 90^\circ$

$\therefore$ From the quadrilateral ADME :

$m(\angle DME) = 360^\circ - (90^\circ + 90^\circ + 56^\circ)$

$= 124^\circ$ (The req.)

7 New Valley

1

1 d 2 a 3 c

4 d 5 b 6 c

2

[a] $\because m(\overset{\frown}{BC}) = 2\, m(\angle CAB) = 2 \times 30^\circ = 60^\circ$ (1)

, $\overline{AB}$ is a diameter

, D is the midpoint of $\overset{\frown}{AC}$

$\therefore m(\overset{\frown}{AD}) = m(\overset{\frown}{CD}) = \frac{180^\circ - 60^\circ}{2} = 60^\circ$ (2)

(First req.)

From (1) and (2) : $\therefore m(\overset{\frown}{AD}) = m(\overset{\frown}{BC})$

$\therefore m(\angle ACD) = m(\angle CAB)$ and they are alternate angles

$\therefore \overline{AB} \,//\, \overline{CD}$ (Second req.)

[b] $\because$ BCDE is a cyclic quadrilateral

$\therefore m(\angle CBE) = 180^\circ - 125^\circ = 55^\circ$

, $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore AB = AC$

$\therefore$ In ΔABC : $m(\angle ACB) = m(\angle ABC)$

$= \frac{180^\circ - 70^\circ}{2} = 55^\circ$

$\therefore m(\angle BEC)$ (inscribed) $= m(\angle ACB)$ (tangency)

$= 55^\circ$

$\therefore m(\angle CBE) = m(\angle BEC)$

$\therefore$ In ΔCBE : $CB = CE$ (Q.E.D. 1)

, $\because m(\angle CBE) = m(\angle ACB) = 55^\circ$

and they are alternate angles

$\therefore \overrightarrow{AC} \,//\, \overline{BE}$ (Q.E.D. 2)

3

[a] State by yourself.

[b] $\because \overline{CD}$ is a diameter

$\therefore m(\angle DXC) = 90^\circ$

, $\because \overline{CD} \perp \overline{AB}$

$\therefore m(\angle YEC) = 90^\circ$

$\therefore m(\angle YXC) + m(\angle YEC) = 90^\circ + 90^\circ = 180^\circ$

$\therefore$ XYEC is a cyclic quadrilateral (Q.E.D. 1)

$\therefore m(\angle DYE) = m(\angle C)$

, $\because m(\angle DBX) = m(\angle C)$

(two inscribed angles subtended by $\overset{\frown}{XD}$)

$\therefore m(\angle DYB) = m(\angle DBX)$ (Q.E.D. 2)

4

[a] $\because \overline{CF}$, $\overline{CE}$ are two tangent-segments to the circle

$\therefore CF = CE = 3$ cm.

$\because \overline{AD}$, $\overline{AF}$ are two tangent-segments to the circle.

$\therefore AD = AF$

$\because \overline{BD}$, $\overline{BE}$ are two tangent-segments to the circle.

$\therefore BD = BE$

, $\because AD + BD = 9$ cm. $\therefore AF + BE = 9$ cm.

$\therefore$ The perimeter of ΔABC

$= 9 + 9 + 3 + 3 = 24$ cm. (The req.)

[b] $\because m(\angle ACB) = \frac{1}{2}\, m(\angle AMB) = \frac{1}{2} \times 120^\circ = 60^\circ$ (1)

(inscribed and central angles subtended by $\overset{\frown}{AB}$)

, $\because \overleftrightarrow{CD} \,//\, \overline{AB}$ $\therefore m(\overset{\frown}{CA}) = m(\overset{\frown}{CB})$

$\therefore CA = CB$ (2)

From (1) and (2) :

$\therefore \Delta ABC$ is an equilateral triangle. (Q.E.D.)

5

[a] $\because MF = ME = r$, $XF = YE$

Subtracting : $\therefore MX = MY$

, $\because \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{CD}$

$\therefore AB = CD$ (Q.E.D. 1)

$\because \overline{MX} \perp \overline{AB}$ $\therefore$ X is the midpoint of $\overline{AB}$

$\therefore AX = \frac{1}{2} AB$

, $\because \overline{MY} \perp \overline{CD}$ $\therefore$ Y is the midpoint of $\overline{CD}$

$\therefore CY = \frac{1}{2} CD$

, $\because AB = CD$ $\therefore AX = CY$

$\therefore$ In $\Delta\Delta$ AXF , CYE : $\begin{cases} AX = CY \\ XF = YE \\ m(\angle AXF) = m(\angle CYE) \end{cases}$

$\therefore \Delta AXF \equiv \Delta CYE$

$\therefore AF = CE$ (Q.E.D. 2)

[b] In Δ AMB : $\because AM = BM = r$

$\therefore m(\angle MBA) = m(\angle MAB) = 20^\circ$

, $\because$ C is the midpoint of $\overline{AB}$

$\therefore \overline{MC} \perp \overline{AB}$ $\therefore m(\angle MCB) = 90^\circ$

$\therefore$ In Δ BCM :

$m(\angle BMC) = 180^\circ - (90^\circ + 20^\circ) = 70^\circ$

$\therefore m(\angle BED) = \frac{1}{2} m(\angle BMD) = \frac{1}{2} \times 70^\circ = 35^\circ$

(inscribed and central angles subtended by $\overset{\frown}{BD}$)

(First req.)

$\because m(\angle MAB) = m(\angle MBA) = 20^\circ$

$\therefore m(\angle AMB) = 180^\circ - (20^\circ + 20^\circ) = 140^\circ$

$\therefore m(\overset{\frown}{ADB}) = m(\angle AMB) = 140^\circ$ (Second req.)

8 South Sinai

1 (1) b (2) b (3) c

(4) d (5) a (6) d

2

[a] $m(\angle BAD) = \frac{1}{2} m(\angle BMD) = \frac{1}{2} \times 150^\circ = 75^\circ$

(inscribed and central angles subtended by $\overset{\frown}{BD}$)

(First req.)

$\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle BCD) = 180^\circ - 75^\circ = 105^\circ$ (Second req.)

[b] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle.

$\therefore AB = AC$ $\therefore 2x - 1 = x + 2$

$\therefore x = 3$ cm. (First req.)

$\therefore AB = AC = 5$ cm. and $BC = 4$ cm.

$\therefore$ The perimeter of $\Delta ABC = 5 + 5 + 4 = 14$ cm.

(Second req.)

3

[a] State by yourself.

[b] In Δ ABD : $\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB) = 30^\circ$

$\therefore m(\angle A) = 180^\circ - 2 \times 30^\circ = 120^\circ$

$\therefore m(\angle A) + m(\angle C) = 120^\circ + 60^\circ = 180^\circ$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

4

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\therefore m(\angle MXA) = 90^\circ$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$

$\therefore m(\angle MYA) = 90^\circ$

From the quadrilateral AXMY :

$m(\angle DME) = 360^\circ - (90^\circ + 90^\circ + 70^\circ) = 110^\circ$

(First req.)

, $\because AB = AC$, $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

$\therefore MX = MY$

, $\because MD = ME = r$

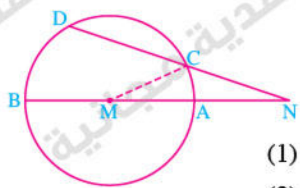
By subtracting : $\therefore XD = YE$ (Second req.)

[b] $\because m(\angle CAE) = 180^\circ - 130^\circ = 50^\circ$

$\therefore m(\angle ABC)$ (inscribed) $= m(\angle CAE)$ (tangency)

$= 50^\circ$ (The req.)

5



[a] Const. : Draw $\overline{MC}$

Proof : In Δ NMC :

$\because CN + MC > NM$ (1)

, $MC = MA = r$ (2)

Subtracting (2) from (1) :

$\therefore CN > AN$ (Q.E.D.)

[b] $\because \overline{DE} \,/\!/\, \overline{BC}$ $\therefore m(\overset{\frown}{DB}) = m(\overset{\frown}{EC})$

$\therefore m(\angle DAB) = m(\angle EAC)$

Adding $m(\angle BAC)$ to both sides

$\therefore m(\angle DAC) = m(\angle BAE)$ (Q.E.D.)

9 North Sinai

1 1 b 2 b 3 c 4 a 5 a 6 c

2

[a] $\because$ $\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M

$\therefore AB = AC$ $\therefore 2x - 3 = 15$

$\therefore 2x = 18$ $\therefore x = 9$ cm.

, $\because$ $\overline{AC}$, $\overline{AD}$ are two tangent-segments to the circle N

$\therefore AC = AD$

$\therefore y - 2 = 15$ $\therefore y = 17$ cm. (The req.)

[b] $\because$ X is the midpoint of $\overline{AB}$ $\therefore \overline{MX} \perp \overline{AB}$

, $\because$ Y is the midpoint of $\overline{AC}$ $\therefore \overline{MY} \perp \overline{AC}$

, $\because AB = AC$ $\therefore MX = MY$

, $\because MD = ME = r$ $\therefore XD = YE$

(Q.E.D.)

3

[a] $\because m(\widehat{BC}) = m(\angle BMC) = 60°$

$\therefore m(\widehat{BD}) = m(\widehat{BC}) = 60°$

$\therefore m(\angle BAD) = \frac{1}{2} m(\widehat{BD}) = \frac{1}{2} \times 60° = 30°$

(The req.)

[b] $\because$ $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle at B and C

$\therefore m(\angle ABC) = m(\angle ACB)$

, $\because m(\angle ABC)$ (tangency)

$= m(\angle BDC)$ (inscribed) $= 65°$

$\therefore m(\angle BAC) = 180° - (65° + 65°) = 50°$ (The req.)

4

[a] $\because$ E is the midpoint of $\overline{BC}$

$\therefore \overline{ME} \perp \overline{BC}$ $\therefore m(\angle MEA) = 90°$

, $\because \overrightarrow{AD}$ is a tangent to the circle at D

$\therefore \overline{MD} \perp \overrightarrow{AD}$ $\therefore m(\angle MDA) = 90°$

$\therefore$ From the quadrilateral ADME :

$m(\angle DME) = 360° - (90° + 90° + 70°) = 110°$

(The req.)

[b] $\because$ $\overline{BC}$ is a diameter

$\therefore m(\angle BAC) = 90°$

$\therefore m(\angle BAE) + m(\angle EDB) = 90° + 90° = 180°$

$\therefore$ ABDE is a cyclic quadrilateral. (Q.E.D. 1)

$\therefore m(\angle CED) = m(\angle ABC)$

, $\because m(\angle ABC) = \frac{1}{2} m(\widehat{AC})$

$\therefore m(\angle CED) = \frac{1}{2} m(\widehat{AC})$ (Q.E.D. 2)

5

[a] $m(\angle BAC) = \frac{1}{2} m(\angle BMC) = \frac{1}{2} \times 120° = 60°$

(inscribed and central angles subtended by $\widehat{BC}$)

(First req.)

, $\because$ ABDC is a cyclic quadrilateral.

$\therefore m(\angle BDC) = 180° - 60° = 120°$ (Second req.)

[b] $\because m(\angle A) = \frac{1}{2}\left[m(\widehat{CE}) - m(\widehat{BD})\right]$

$\therefore 30° = \frac{1}{2}\left[100° - m(\widehat{BD})\right]$

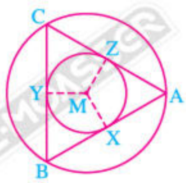
$\therefore 60° = 100° - m(\widehat{BD})$

$\therefore m(\widehat{BD}) = 100° - 60° = 40°$ (The req.)

10 Red Sea

1 1 b 2 a 3 d 4 c 5 c 6 b

2



[a] Const. : Draw $\overline{MX}$, $\overline{MY}$, $\overline{MZ}$

Proof : $\because$ $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ are tangent-segments to the smaller circle at X , Y , Z

$\therefore \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{BC}$, $\overline{MZ} \perp \overline{AC}$

, $\because MX = MY = MZ$ (radii lengths of the smaller circle)

$\therefore AB = BC = AC$

$\therefore \Delta ABC$ is an equilateral triangle. (Q.E.D.)

[b] $m(\angle E) = m(\angle C) = 20°$

(two inscribed angles subtended by $\widehat{BD}$) (First req.)

$\because \angle CDE$ is an exterior angle of ΔACD

$\therefore m(\angle CDE) = 43° + 20° = 63°$

$\therefore m(\widehat{CE}) = 2\, m(\angle CDE) = 2 \times 63° = 126°$

(Second req.)

3

[a] $\because$ E is the midpoint of $\overline{BC}$

$\therefore \overline{ME} \perp \overline{BC}$ $\therefore m(\angle MEA) = 90°$

, $\because \overrightarrow{AD}$ is a tangent to the circle at D

$\therefore \overline{MD} \perp \overrightarrow{AD}$ $\therefore$ m ($\angle$ MDA) = 90°

$\therefore$ From the quadrilateral ADME :

m ($\angle$ M) = 360° – (90° + 90° + 56°) = 124° (The req.)

[b] m ($\angle$ B) = $\frac{1}{2}$ m ($\angle$ M) = $\frac{1}{2}$ × 60° = 30°

(inscribed and central angles subtended by $\overset{\frown}{AC}$)

(First req.)

, $\because \overline{MC} // \overline{BA}$, $\overleftrightarrow{MA}$ is a transversal

$\therefore$ m ($\angle$ A) = m ($\angle$ M) = 60° (alternate angles)

$\therefore$ In Δ BEA : m ($\angle$ BEA) = 180° – (30° + 60°) = 90°

$\therefore \overline{BC} \perp \overline{AM}$ (Second req.)

4

[a] $\because \overline{XY} // \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m ($\angle$ AYX) = m ($\angle$ C) (corresponding angles)

, $\because$ m ($\angle$ C) (inscribed) = m ($\angle$ BAD) (tangency)

$\therefore$ m ($\angle$ AYX) = m ($\angle$ XAD)

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] In Δ ABD : $\because$ AB = AD

$\therefore$ m ($\angle$ ABD) = m ($\angle$ ADB) = 30°

$\therefore$ m ($\angle$ BAD) = 180° – 2 × 30° = 120°

$\therefore$ m ($\angle$ BAD) = m ($\angle$ DCE)

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

5

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ A) = 180° – 120° = 60° (First req.)

, $\because \overline{AB}$ is a diameter

$\therefore$ m ($\angle$ ADB) = 90°

$\therefore$ In Δ ADB : m ($\angle$ ABD) = 180° – (90° + 60°) = 30°

(Second req.)

[b] $\because$ m ($\angle$ BCD) = $\frac{1}{2}$ m ($\angle$ M) = $\frac{1}{2}$ × 130° = 65°

(inscribed and central angles subtended by $\overset{\frown}{BD}$)

, $\because \overline{AB} // \overline{CD}$, $\overleftrightarrow{BC}$ is a transversal.

$\therefore$ m ($\angle$ ABC) = m ($\angle$ BCD) = 65° (alternate angles)

, $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle.

$\therefore$ AB = AC

$\therefore$ In Δ ABC : ($\angle$ ACB) = m ($\angle$ ABC) = 65°

$\therefore$ m ($\angle$ A) = 180° – 2 × 65° = 50° (The req.)

امتحانات رقم (3)

الترم الثانى

1 Cairo Governorate

Answer the following questions : (Calculator is allowed)

1 Choose the correct answer from those given :

1 The parallelogram in which the two diagonals are equal in length is called ....

(a) a square. (b) a rhombus. (c) a rectangle. (d) a trapezium.

2 The chord whose length is 8 cm. in a circle of radius length 5 cm. is at .... cm. distant from its centre.

(a) 3 (b) 4 (c) 5 (d) 10

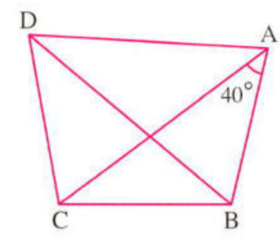
3 The area of the rectangle whose dimensions are 4 cm. and 6 cm. equals .... cm^2.

(a) 10 (b) 20 (c) 24 (d) 48

4 If two chords intersected inside the circle , then the measure of the angle between them equals .... the sum of measures of the two opposite arcs.

(a) double (b) half (c) third (d) quarter

5 **In the opposite figure :**



ABCD is a cyclic quadrilateral.

If m (∠ CAB) = 40° , then m (∠ CDB) = ...

(a) 20° (b) 40°

(c) 80° (d) 140°

6 The point of concurrence of the medians of the triangle divides each median by the ratio .... from the base.

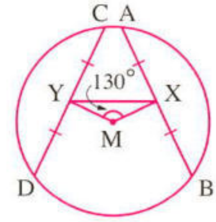
(a) 1 : 3 (b) 2 : 1 (c) 3 : 1 (d) 1 : 2

2 [a] If M and N are two circles , the lengths of their radii are 8 cm. and 6 cm. respectively , **, find MN in each of the following cases :**

1 The two circles are touching externally.

2 The two circles are touching internally.

[b] **In the opposite figure :**

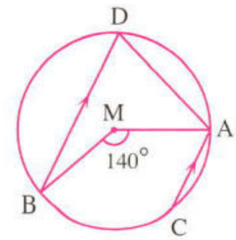


In the circle of centre M , AB = CD ,

m (∠ XMY) = 130° , X and Y are the midpoints of $\overline{AB}$ and $\overline{CD}$

Find with proof : m (∠ MXY)

3 [a] **In the opposite figure :**



In the circle M , $\overline{DB} \parallel \overline{AC}$,

m (∠ AMB) = 140°

Find with proof :

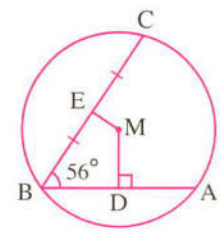
1 m (∠ ADB)

2 m (∠ CAD)

[b] Find the length of the arc opposite to a central angle of measure 120° in a circle of circumference 132 cm.

---

4 [a] **In the opposite figure :**



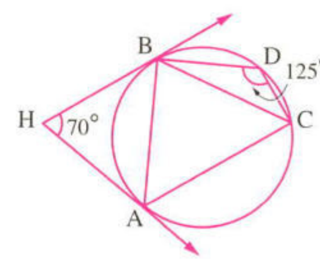
$\overline{AB}$ and $\overline{BC}$ are two chords in the circle M ,

$\overline{MD} \perp \overline{AB}$, E is the midpoint of $\overline{BC}$, m (∠ ABC) = 56°

1 **Prove that :** the figure EMDB is a cyclic quadrilateral.

2 **Find with proof :** m (∠ EMD)

[b] **In the opposite figure :**



$\overrightarrow{HA}$ and $\overrightarrow{HB}$ are two tangents to the circle at A and B ,

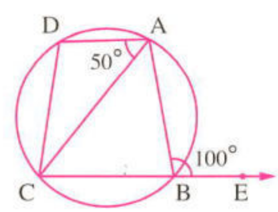
m (∠ BDC) = 125° , m (∠ AHB) = 70°

1 **Find with proof :** m (∠ ACB)

2 **Prove that :** BA = BC

---

5 [a] **In the opposite figure :**



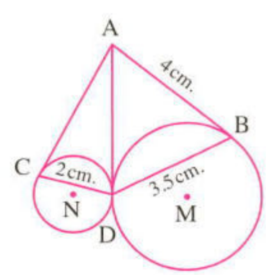
E ∈ $\overrightarrow{CB}$, m (∠ ABE) = 100°

, m (∠ CAD) = 50°

Find with proof :

1 m (∠ CDA) 2 m ($\overset{\frown}{DA}$)

[b] **In the opposite figure :**



M and N are two circles touching externally at D ,

$\overline{AB}$ touches the circle M at B ,

$\overline{AC}$ touches the circle N at C ,

AB = 4 cm. , BD = 3.5 cm. and DC = 2 cm.

Find with proof :

The perimeter of the figure ABDC

2 Giza Governorate

Answer the following questions :

1 Choose the correct answer :

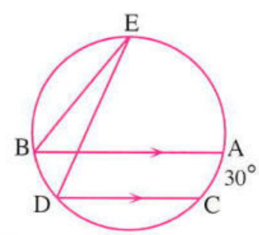
1 If M is a circle with radius length 7 cm. , A is a point in the plane of the circle , MA = 4 cm. , then A lies .... the circle M

(a) inside (b) outside (c) on (d) at the centre of

2 If the projection of a line segment on a straight line is a point , then the line segment is .... the straight line.

(a) // (b) $\perp$ (c) $\in$ (d) $\subset$

3 **In the opposite figure :**



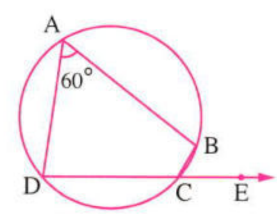
$\overline{AB}$ and $\overline{CD}$ are two parallel chords , m $(\overset{\frown}{AC})$ = 30° , then m (∠ BED) = ...

(a) 15° (b) 10° (c) 30° (d) 60°

4 If the side length of a rhombus is L cm. , then its perimeter = ... cm.

(a) L^2 (b) $2\,L^2$ (c) 4 L (d) $2\sqrt{2}$ L

5 **In the opposite figure :**



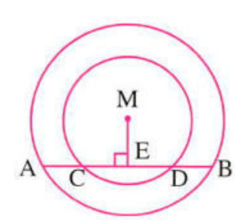
If m (∠ BAD) = 60° , then m (∠ BCE) = ...

(a) 30° (b) 60° (c) 80° (d) 120°

6 If ABC is a right-angled triangle at B , then AC .... BC

(a) < (b) > (c) = (d) is twice

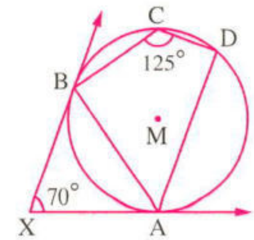
2 [a] In the opposite figure :



Two concentric circles with centre M , $\overline{AB}$ is a chord of the greater circle and intersects the smaller circle at C , D , $\overline{ME} \perp \overline{AB}$

Prove that : AC = BD

[b] In the opposite figure :



$\overrightarrow{XA}$ and $\overrightarrow{XB}$ are two tangents to the circle at A and B ,

m (∠ AXB) = 70° and m (∠ DCB) = 125°

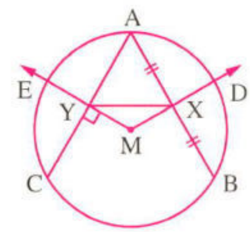
Prove that :

1 $\overrightarrow{AB}$ bisects ∠ DAX

2 $\overline{AD}$ // $\overrightarrow{XB}$

---

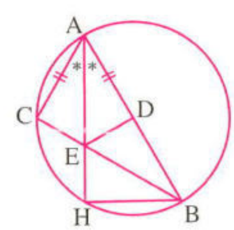
3 **[a] In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X is the midpoint of $\overline{AB}$, $\overrightarrow{MX}$ intersects the circle at D and $\overrightarrow{MY} \perp \overline{AC}$ intersecting it at Y and intersecting the circle at E

Prove that : 1 m (∠ MXY) = m (∠ MYX) 2 XD = YE

[b] In the opposite figure :



Δ ABC is an inscribed triangle inside a circle , AB > AC

, D ∈ $\overline{AB}$, AC = AD

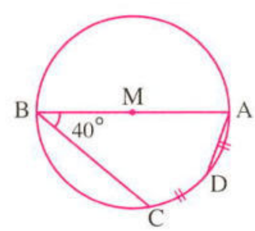
, $\overrightarrow{AE}$ bisects ∠ BAC intersecting $\overline{BC}$ at E

and intersecting the circle at H

Prove that : The figure BDEH is a cyclic quadrilateral.

---

4 **[a] In the opposite figure :**



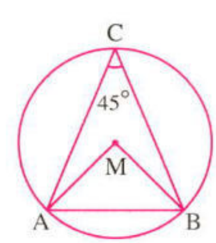
$\overline{AB}$ is a diameter of the circle M ,

m (∠ ABC) = 40° ,

D is the midpoint of $\overset{\frown}{AC}$

Find : m (∠ BAD)

[b] In the opposite figure :



m (∠ ACB) = 45°

Find : m (∠ MAB)

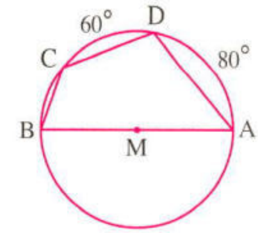
5 **[a] In the opposite figure :**

m $(\overset{\frown}{AD}) = 80°$,

m $(\overset{\frown}{DC}) = 60°$,

$\overline{AB}$ is a diameter of the circle M

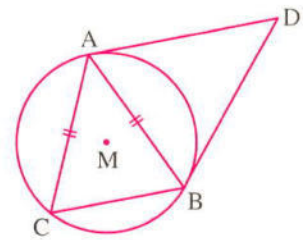
Find the measure of the interior angles of the figure ABCD



[b] In the opposite figure :

$\overline{DA}$ and $\overline{DB}$ are two tangent-segments to the circle M and AB = AC

Prove that : $\overleftrightarrow{AC}$ is a tangent to the circumcircle of Δ ABD



3 Alexandria Governorate

Answer the following questions : *(Calculators are permitted)*

1 **Choose the correct answer from those given :**

1 The measure of the inscribed angle equals .... the measure of the central angle subtended by the same arc.

(a) half (b) twice (c) quarter (d) third

2 The number of circles which pass through three non-collinear points is ....

(a) zero (b) 1 (c) 2 (d) 3

3 M and N are two touching externally circles of radii lengths 5 cm. and 9 cm. respectively , then MN = ... cm.

(a) 4 (b) 5 (c) 9 (d) 14

4 In the triangle ABC , if m $(\angle B) = 90°$, then $(AB)^2$ = ...

(a) $(AC)^2 + (BC)^2$ (b) $(AC)^2 - (BC)^2$ (c) $(BC)^2 - (AC)^2$ (d) $\frac{AB \times BC}{AC}$

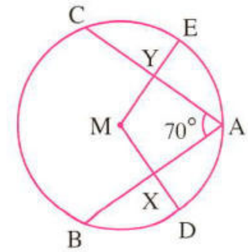
5 ABC is an isosceles triangle in which AB = AC , m $(\angle A) = 70°$, then m $(\angle B)$ = ...

(a) 70° (b) 55° (c) 65° (d) 110°

6 A square of side length 5 cm. , then its surface area = ... cm^2.

(a) 10 (b) 20 (c) 25 (d) 125

2 [a] **In the opposite figure :**

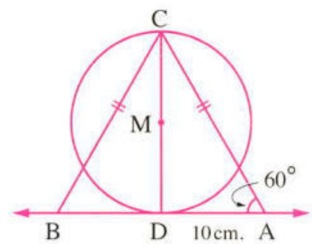


$\overline{AB}$, $\overline{AC}$ are two chords in the circle M , X , Y are the midpoints of $\overline{AB}$, $\overline{AC}$ respectively , m (∠ CAB) = 70°

1 **Find :** m (∠ DME)

2 If XD = YE **, prove that :** AB = AC

[b] **In the opposite figure :**



$\overline{DC}$ is a diameter of the circle M , AC = BC ,

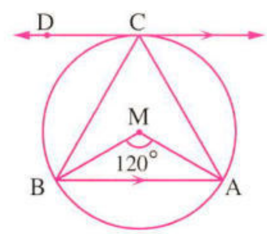
m (∠ CAB) = 60° , AD = 10 cm. ,

$\overleftrightarrow{AB}$ is a tangent to the circle at D

Find : The perimeter of the triangle ABC

---

3 [a] **In the opposite figure :**



M is the circumcircle of Δ ABC ,

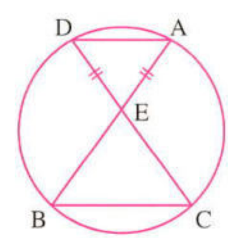
m (∠ AMB) = 120° ,

$\overleftrightarrow{CD}$ is a tangent to the circle at C ,

$\overleftrightarrow{CD}$ // $\overline{AB}$

Prove that : Δ ABC is an equilateral triangle.

[b] **In the opposite figure :**

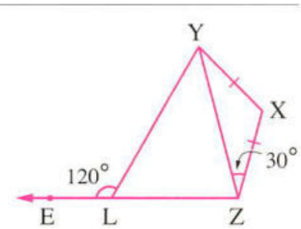


$\overline{AB} \cap \overline{CD} = \{E\}$, EA = ED

Prove that : EB = EC

---

4 [a] **In the opposite figure :**

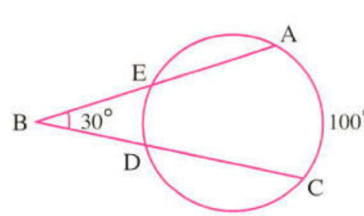


XY = XZ , m (∠ XZY) = 30° ,

m (∠ ELY) = 120° where E ∈ $\overrightarrow{ZL}$

Prove that : YXZL is a cyclic quadrilateral.

[b] **In the opposite figure :**



If m $(\overset{\frown}{AC})$ = 100° ,

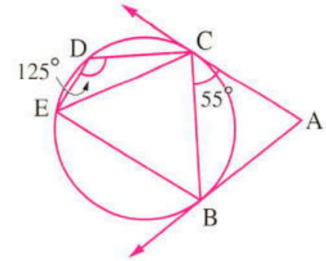
m (∠ B) = 30°

, find with proof : m $(\overset{\frown}{DE})$

5 **[a] In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are tangents to the circle at B , C

, m (∠ ACB) = 55° , m (∠ CDE) = 125°



1 **Find :** m (∠ A)

2 **Prove that :** CB = CE

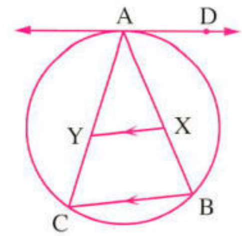
[b] In the opposite figure :

ABC is a triangle inscribed in a circle ,

$\overleftrightarrow{AD}$ is a tangent to the circle at A , $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent

to the circle passing through the points A , X , Y



4 El-Kalyoubia Governorate

Answer the following questions :

1 **Choose the correct answer from the given ones :**

1 In Δ ABC , if $(AB)^2 + (BC)^2 = (AC)^2$, then ∠ B is ……………

(a) obtuse. (b) right. (c) acute. (d) straight.

2 The number of circles which pass through three non-collinear points is ……………

(a) 0 (b) 1 (c) 2 (d) infinite.

3 If ABCD is a cyclic quadrilateral , then m (∠ A) + m (∠ C) – 50° = ……………

(a) 180° (b) 150° (c) 130° (d) 100°

4 If M and N are two circles the lengths of their radii are 5 cm. and 7 cm. and MN = 12 cm. , then the two circles are ……………

(a) distant. (b) touching externally.

(c) touching internally. (d) concentric.

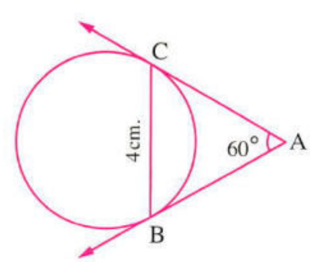
5 The inscribed angle drawn in quarter of a circle is …………… angle.

(a) an acute (b) a right (c) a straight (d) an obtuse

6 **In the opposite figure :**

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents ,

then the perimeter of Δ ABC = …………… cm.



(a) 4 (b) 8

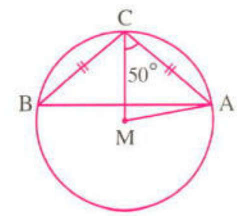
(c) 10 (d) 12

2 [a] **In the opposite figure :**

In the circle M , AC = BC ,

m (∠ ACM) = 50°

Find : m (∠ CAB)



[b] **In the opposite figure :**

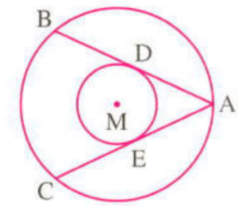
Two concentric circles

of centre M , $\overline{AB}$ and $\overline{AC}$

are two tangent-segments to the small

circle at D and E

Prove that : AB = AC



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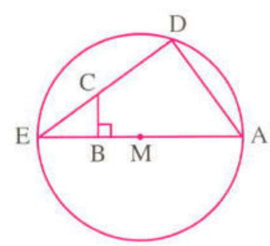
3 [a] **In the opposite figure :**

$\overline{AE}$ is a diameter

in the circle M , $\overline{CB} \perp \overline{AE}$

Prove that : the figure.

ABCD is a cyclic quadrilateral.



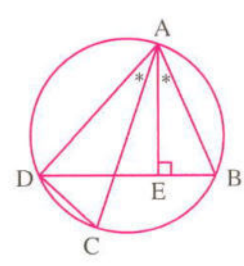
[b] **In the opposite figure :**

m (∠ BAE) = m (∠ DAC) ,

$\overline{AE} \perp \overline{BD}$

Prove that :

$\overline{AC}$ is a diameter in the circle.



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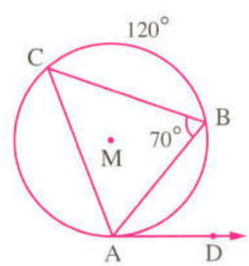
4 [a] **In the opposite figure :**

$\overrightarrow{AD}$ is a tangent to the circle M ,

m (∠ B) = 70° ,

m $(\overset{\frown}{BC})$ = 120°

Find with proof : m (∠ DAB)



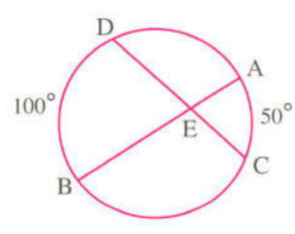
[b] **In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{E\}$,

m $(\overset{\frown}{AC})$ = 50° ,

m $(\overset{\frown}{DB})$ = 100°

Find : m (∠ AEC)

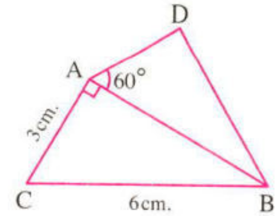


5 [a] **In the opposite figure :**

m (∠ DAB) = 60° , m (∠ BAC) = 90° ,

AC = 3 cm. , BC = 6 cm.

Prove that : $\overrightarrow{AD}$ is a tangent to the circle which passes through the vertices of Δ ABC



[b] **In the opposite figure :**

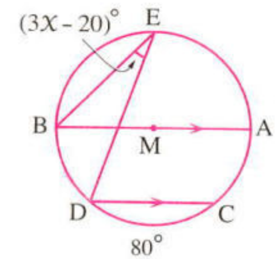
$\overline{AB}$ // $\overline{CD}$

where $\overline{AB}$ is a diameter in the circle M ,

m ($\overset{\frown}{CD}$) = 80° ,

m (∠ BED) = (3 X – 20)°

Find : The value of X



5 El-Sharkia Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from the given ones :**

1. M , N are two touching externally circles , the lengths of their radii are 7 cm. , 3 cm. , then MN = ... cm.

   (a) 4 (b) 12 (c) 6 (d) 10

2. Δ ABC is right-angled at B , the lengths of sides of the right angle are 6 cm. , 8 cm. , then the area of the circumcircle of Δ ABC = ... π cm^2

   (a) 36 (b) 64 (c) 25 (d) 100

3. ABCD is a cyclic quadrilateral , m (∠ A) = 110° , then m (∠ C) = ...

   (a) 70° (b) 250° (c) 80° (d) 110°

4. Lengths of two adjacent sides of a parallelogram are 7 cm. , 5 cm. , the longer height is 6 cm. , then its area = ... cm^2

   (a) 35 (b) 42 (c) 30 (d) 18

5. The measure of the inscribed angle equals .... the measure of the central angle subtended by the same arc.

   (a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{3}$ (d) 3

6. The circumference of a circle M is 12 π cm. , the point A is in the plane of the circle. If MA = 5 cm. , then the point A lies .... the circle.

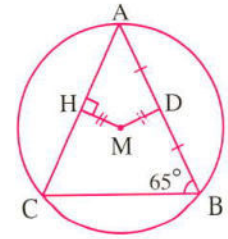
   (a) outside (b) inside (c) on (d) at the centre of

2 **[a] In the opposite figure :**

D is the midpoint of $\overline{AB}$, $\overline{MH} \perp \overline{AC}$,

MD = MH , m ($\angle$ B) = 65°

Find : m ($\angle$ A)



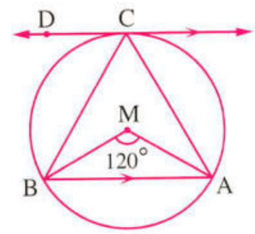
[b] In the opposite figure :

A circle M , $\overleftrightarrow{CD}$ is a tangent at C

, $\overleftrightarrow{CD}$ // $\overline{AB}$

, m ($\angle$ AMB) = 120°

Prove that : Δ CAB is an equilateral triangle.



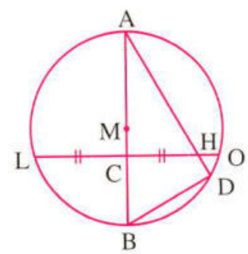
3 **[a] In the opposite figure :**

A circle M , $\overline{AB}$ is a diameter ,

C is the midpoint of $\overline{OL}$

Prove that :

The figure HDBC is a cyclic quadrilateral.



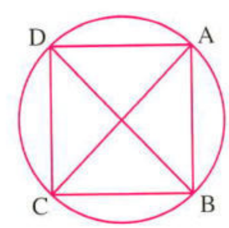
[b] In the opposite figure :

ABCD is a cyclic quadrilateral

where AC = BD , AB = (3 X – 5) cm. ,

CD = (X + 3) cm.

Find : The length of $\overline{AB}$



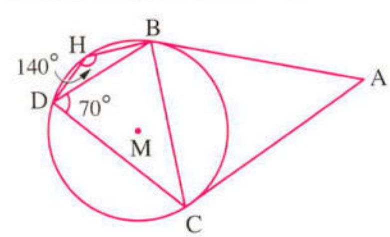
4 **[a] In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle M

at B , C , m ($\angle$ H) = 140° , m ($\angle$ BDC) = 70°

1 **Find :** m ($\angle$ A)

2 **Prove that :** $\overleftrightarrow{CD}$ is a tangent to the circle which passes through the points A , B and C

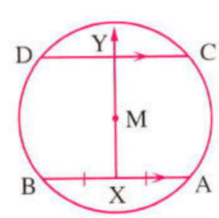


[b] In the opposite figure :

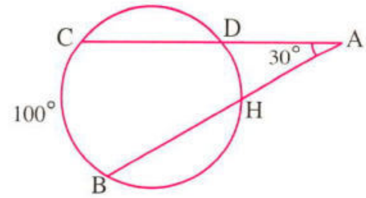
A circle M where X is the midpoint of $\overline{AB}$,

$\overline{AB}$ // $\overline{CD}$, $\overrightarrow{XM} \cap \overline{CD} = \{Y\}$

Prove that : Y is the midpoint of $\overline{CD}$



5 **[a] In the opposite figure :**

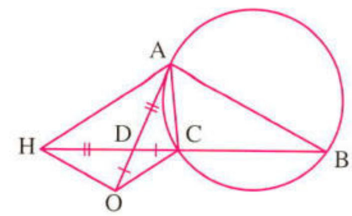


m (∠ A) = 30° ,

m $(\overset{\frown}{BC})$ = 100°

Find : m $(\overset{\frown}{DH})$

[b] In the opposite figure :



$\overline{AD}$ is a tangent-segment to the circle at the point A

, AD = DH , CD = DO

Prove that : 1 ACOH is a cyclic quadrilateral.

2 $\overline{AB}$ // $\overline{OH}$

6 El-Monofia Governorate

Answer the following questions : (Calculator is allowed)

1 **Choose the correct answer from those given :**

1 The number of axes of symmetry of the rhombus equals ....

(a) zero (b) 1 (c) 2 (d) 4

2 The point of concurrence of the medians of the triangle divides the median by the ratio of .... from the base.

(a) 1 : 2 (b) 2 : 1 (c) 1 : 3 (d) 3 : 1

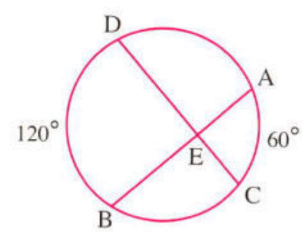
3 In Δ ABC , if $(AB)^2 + (BC)^2 < (AC)^2$, then ∠ C is ....

(a) straight. (b) obtuse. (c) right. (d) acute.

4 If the straight line L is a tangent to the circle M of diameter length 8 cm. , then the distance between L and the centre of the circle equals .... cm.

(a) 3 (b) 4 (c) 6 (d) 8

5 **In the opposite figure :**



If $\overline{AB} \cap \overline{CD} = \{E\}$, m $(\overset{\frown}{AC})$ = 60° ,

m $(\overset{\frown}{BD})$ = 120° ,

then m (∠ AEC) = ...

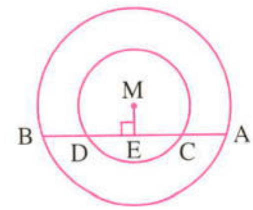
(a) 50° (b) 70°

(c) 90° (d) 180°

6 If ABCD is a cyclic quadrilateral , m (∠ A) : m (∠ C) = 1 : 2 , then m (∠ A) = ...

(a) 30° (b) 60° (c) 90° (d) 120°

2 [a] **In the opposite figure :**



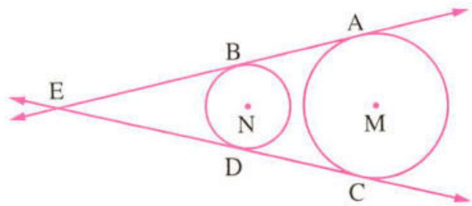
Two concentric circles of centre M

, $\overline{AB}$ is a chord in the greater circle

intersecting the smaller circle at C , D and $\overline{ME} \perp \overline{AB}$

Prove that : AC = BD

[b] **In the opposite figure :**



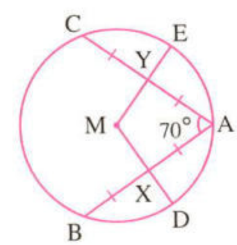
$\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ are the common tangents

to the two circles M , N and $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$

Prove that : AB = CD

---

3 [a] **In the opposite figure :**

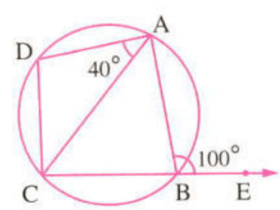


$\overline{AB}$, $\overline{AC}$ are two chords equal in length in the circle M , X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$ and m (∠ BAC) = 70°

1 **Find :** m (∠ DME)

2 **Prove that :** XD = YE

[b] **In the opposite figure :**



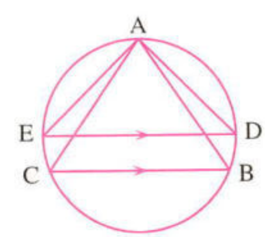
$E \in \overrightarrow{CB}$, m (∠ ABE) = 100°

and m (∠ CAD) = 40°

Prove that : m $(\overparen{AD})$ = m $(\overparen{CD})$

---

4 [a] **In the opposite figure :**

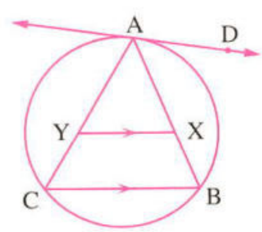


Δ ABC is inscribed in

a circle and $\overline{DE}$ // $\overline{BC}$

Prove that : m (∠ DAC) = m (∠ BAE)

[b] **In the opposite figure :**



Δ ABC is inscribed in a circle ,

$\overleftrightarrow{AD}$ is a tangent to the circle at A ,

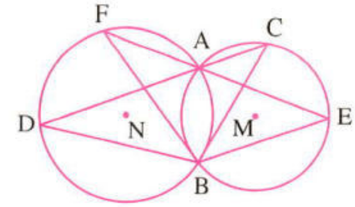
$X \in \overline{AB}$, $Y \in \overline{AC}$

where $\overline{XY}$ // $\overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent

to the circle passing through the points A , X and Y

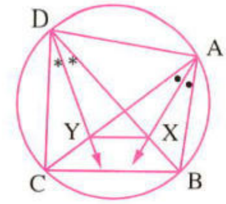
5 [a] **In the opposite figure :**



M and N are two intersecting circles at A and B , $\overleftrightarrow{AC}$ intersects the two circles M , N at C , D respectively , $\overleftrightarrow{AE}$ intersects the two circles M , N at E , F respectively

Prove that : m (∠ CBE) = m (∠ FBD)

[b] **In the opposite figure :**



$\overrightarrow{AX}$ bisects ∠ BAC , $\overrightarrow{DY}$ bisects ∠ BDC

Prove that : AXYD is a cyclic quadrilateral.

7 El-Gharbia Governorate

Answer the following questions :

1 **Choose the correct answer from those given :**

1. The measure of the inscribed angle drawn in a semicircle equals ....

   (a) 120° (b) 180° (c) 108° (d) 90°

2. The slope of the straight line : $3x - 5y = -7$ is ....

   (a) $\frac{3}{5}$ (b) $\frac{-3}{5}$ (c) $\frac{5}{3}$ (d) $\frac{-5}{3}$

3. If ABCD is a cyclic quadrilateral , m (∠ A) = 70° , then m $(\overset{\frown}{BAD})$ = ...

   (a) 35° (b) 110° (c) 140° (d) 220°

4. If ABCD is a rhombus , then the number of circles that pass through the vertices A , B , C is ....

   (a) zero (b) 1 (c) 3 (d) infinite.

5. The square whose side length is 4 cm. , then the length of its diagonal equals .... cm.

   (a) $4\sqrt{2}$ (b) 16 (c) 32 (d) $16\sqrt{2}$

6. The number of altitudes of the isosceles triangle is ....

   (a) zero (b) 1 (c) 3 (d) infinite.

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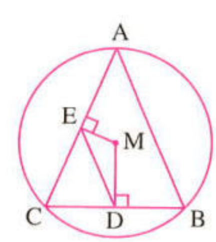
2 [a] **In the opposite figure :**



ABC is a triangle drawn inside a circle with centre M (inscribed triangle) ,

$\overline{MD} \perp \overline{BC}$ and $\overline{ME} \perp \overline{AC}$

Prove that : The perimeter of Δ CDE = $\frac{1}{2}$ the perimeter of Δ ABC

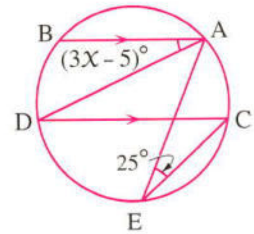
[b] In the opposite figure :

$\overline{AB}$, $\overline{CD}$ are two parallel chords in a circle ,

m (∠ BAD) = $(3x - 5)°$

, m (∠ AEC) = 25°

Find : The value of x



3 **[a] In the opposite figure :**

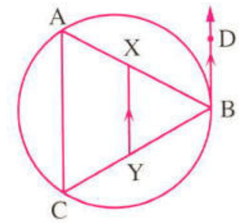
Δ ABC is inscribed in a circle

, $\overrightarrow{BD}$ is a tangent to the circle at B ,

X ∈ $\overline{AB}$, Y ∈ $\overline{BC}$

such that : $\overline{XY}$ // $\overrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.

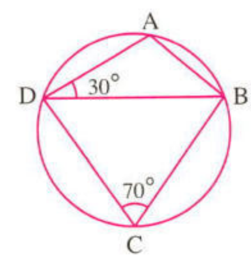


[b] In the opposite figure :

ABCD is a cyclic quadrilateral ,

m (∠ ADB) = 30° , m (∠ BCD) = 70°

Find : m (∠ ABD)

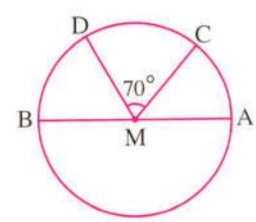


4 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M ,

m (∠ CMD) = 70° , m $(\overset{\frown}{AC})$: m $(\overset{\frown}{BD})$ = 5 : 6

Find : m $(\overset{\frown}{ACD})$



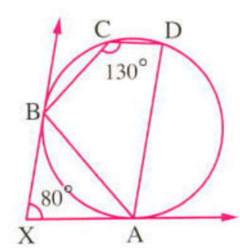
[b] In the opposite figure :

$\overrightarrow{XA}$, $\overrightarrow{XB}$ are two tangents

to the circle at A and B , m (∠ AXB) = 80° ,

m (∠ DCB) = 130°

Prove that : $\overline{AD}$ // $\overrightarrow{XB}$



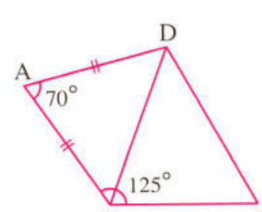
5 **[a] In the opposite figure :**

AB = AD , m (∠ A) = 70° ,

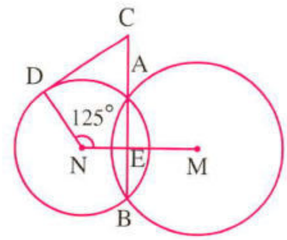
m (∠ ABC) = 125°

Prove that : $\overline{BC}$ is a tangent-segment to

the circle passing through the points A , B and D



[b] In the opposite figure :



M , N are two intersecting circles at A , B ,

$\overline{CD}$ is a tangent-segment to the circle N ,

$C \in \overrightarrow{BA}$, m ($\angle$ MND) = 125°

Find : m ($\angle$ DCE)

8 El-Dakahlia Governorate

Answer the following questions : *(Calculator is permitted)*

1 [a] Choose the correct answer :

1 The circumference of the circle whose diameter length is 6 cm. equals .... cm.

(a) 3 π (b) 6 π (c) 9 π (d) 12 π

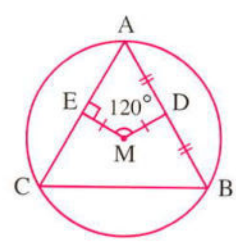
2 The radius length of the circle whose centre is the origin point and passes through the point (3 , 4) equals .... length unit.

(a) 3 (b) 4 (c) 5 (d) 7

3 The inscribed angle which is subtended by a minor arc in a circle is .... angle.

(a) a reflex (b) a right (c) an obtuse (d) an acute

[b] In the opposite figure :



The triangle ABC is an inscribed triangle inside a circle M , D is the midpoint of $\overline{AB}$

, $\overline{ME} \perp \overline{AC}$, MD = ME

, m ($\angle$ DME) = 120°

Prove that : Δ ABC is an equilateral triangle.

2 [a] Choose the correct answer :

1 If $\overline{AB}$ is a line segment of length 4 cm. , then the radius length of the smallest circle which passes through the two points A , B equals .... cm.

(a) 2 (b) 3 (c) 4 (d) 5

2 It is impossible to draw a circle passing through the vertices of ....

(a) a rectangle. (b) a square.

(c) an isosceles trapezium. (d) a parallelogram.

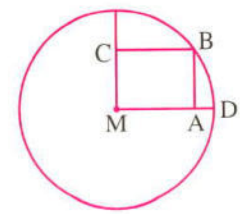
3 **In the opposite figure :**

A circle M of radius length 5 cm. ,

the figure MABC is a rectangle ,

MC = 3 cm. ,

then AD = ... cm.



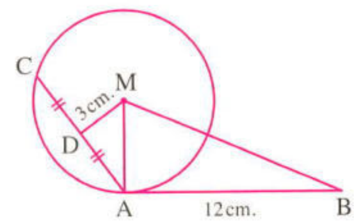
(a) 1 (b) 2 (c) 3 (d) 4

[b] In the opposite figure :

A circle M of radius length 5 cm. , $\overrightarrow{AB}$ is a tangent to the circle at A , D is the mipoint of $\overline{AC}$,

AB = 12 cm. , MD = 3 cm.

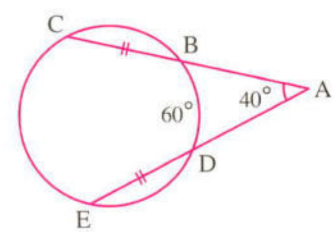
Find : The area of the figure ABMD



3 **[a] In the opposite figure :**

$\overline{CB}$, $\overline{ED}$ are two chords equal in length in a circle where $\overrightarrow{CB} \cap \overrightarrow{ED} = \{A\}$, m (∠ A) = 40° ,

m $(\widehat{BD})$ = 60° **Find :** m $(\widehat{BC})$



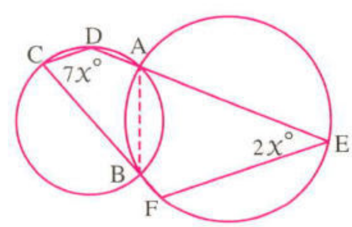
[b] In the opposite figure :

Two intersecting circles at A , B ,

A ∈ $\overline{ED}$, B ∈ $\overline{FC}$, m (∠ E) = $(2X)°$,

m (∠ D) = $(7X)°$

Find : The value of X

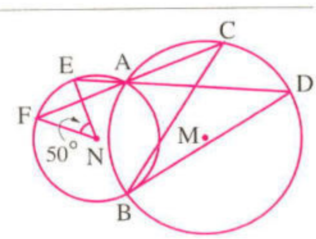


4 **[a] In the opposite figure :**

Two intersecting circles at A , B ,

$\overline{CF} \cap \overline{DE} = \{A\}$, m (∠ ENF) = 50°

Find : m (∠ DBC)

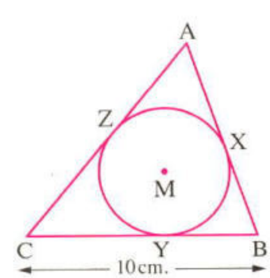


[b] In the opposite figure :

Δ ABC touches the circle M externally at X , Y , Z , the perimeter of Δ ABC = 30 cm.

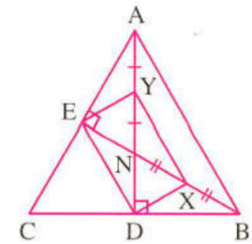
, the length of $\overline{BC}$ = 10 cm.

Find : the length of $\overline{AX}$



5 [a] ABC is a triangle inscribed in a circle , $\overleftrightarrow{AD}$ is a tangent to the circle at A , X ∈ $\overline{AB}$, Y ∈ $\overline{AC}$ where $\overline{XY}$ // $\overline{BC}$ **Prove that :** $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X , Y

[b] **In the opposite figure :**



ABC is a triangle ,

$\overline{AD} \perp \overline{BC}$, $\overline{BE} \perp \overline{AC}$,

X is the midpoint of $\overline{NB}$,

Y is the midpoint of $\overline{NA}$

Prove that : The figure XYED is a cyclic quadrilateral.

9 Ismailia Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer from those given :

1 The measure of the reflex angle of the angle that is measured 60° equals ....

(a) 30° (b) 120° (c) 300° (d) 360°

2 ABCD is a cyclic quadrilateral in which m (∠ A) = m (∠ C) = ...

(a) 60° (b) 90° (c) 120° (d) 180°

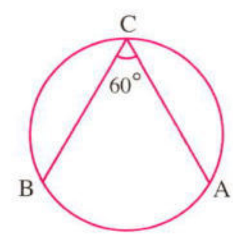
3 The length of the diagonal of a square is 8 cm. , then its surface area = ... cm².

(a) 8 (b) 16 (c) 32 (d) 64

4 Two circles , the lengths of their radii are 3 cm. , 7 cm. , are touching when the distance between their centres ∈ ....

(a)]4 , 10[(b)]10 , ∞[(c)]0 , 4[(d) {4 , 10}

5 **In the opposite figure :**



If m (∠ ACB) = 60° ,

then the length of $\widehat{AB}$ = ...

(a) $\frac{2}{3}\pi r$ (b) $\frac{1}{3}\pi r$

(c) $\frac{1}{6}\pi r$ (d) $2\pi r$

6 The measure of the exterior angle of the equilateral triangle equals ....

(a) 60° (b) 120° (c) 130° (d) 180°

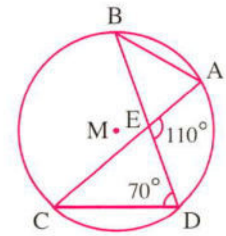
2 [a] **In the opposite figure :**

$\overline{AC}$ and $\overline{BD}$ are two chords in a circle M intersecting at E ,

where m (∠ AED) = 110° ,

m (∠ D) = 70°

Find : m (∠ B)

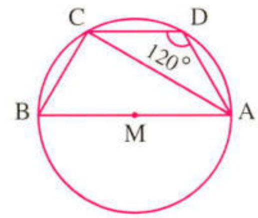


[b] **In the opposite figure :**

$\overline{AB}$ is a diameter of the circle M ,

m (∠ D) = 120°

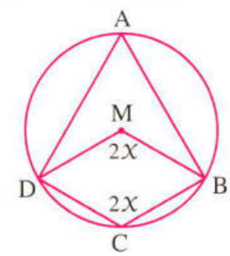
Find : m (∠ CAB)



3 [a] **In the opposite figure :**

m (∠ DMB) = m (∠ DCB) = 2 x

Find : m (∠ A) in degrees.



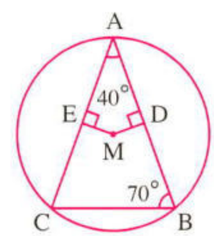
[b] **In the opposite figure :**

ABC is a triangle inscribed in a circle M ,

m (∠ A) = 40° , m (∠ B) = 70° ,

$\overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

Prove that : MD = ME



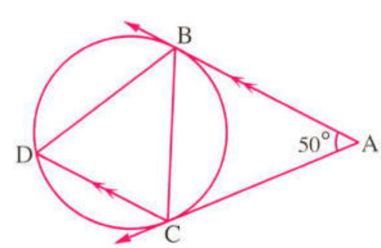
4 [a] **In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C ,

m (∠ A) = 50° , $\overrightarrow{AB}$ // $\overline{CD}$

1 **Find :** m (∠ CDB)

2 **Prove that :** $\overline{BD}$ is a tangent-segment to the circle passing through the vertices of Δ ABC

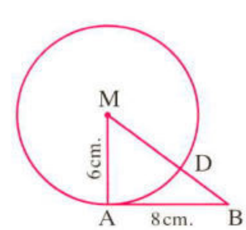


[b] **In the opposite figure :**

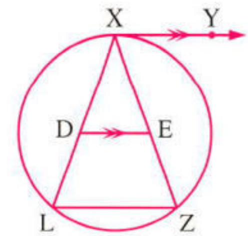
$\overline{AB}$ is a tangent-segment to the circle M ,

AM = 6 cm. , AB = 8 cm.

Find : The length of $\overline{BD}$



5 **[a] In the opposite figure :**

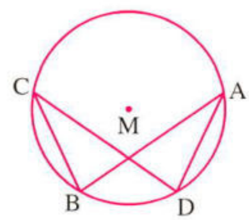


$\overrightarrow{XY}$ is a tangent to the circle ,

$\overrightarrow{XY} \parallel \overline{DE}$

Prove that : EDLZ is a cyclic quadrilateral.

[b] In the opposite figure :



$\overline{AB}$, $\overline{CD}$ are two equal chords in the circle M

Prove that : AD = CB

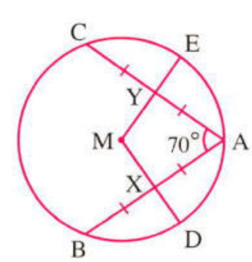
10 Damietta Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from the given ones :**

1. The two diagonals are equal in length and not perpendicular in the ....
   (a) square. (b) rhombus. (c) rectangle. (d) parallelogram.
2. If the side length of a square is $2\sqrt{2}$ cm. , then its area equals .... cm^2.
   (a) $8\sqrt{2}$ (b) $4\sqrt{2}$ (c) 8 (d) 4
3. ABCD is a cyclic quadrilateral , m ($\angle$ B) = 70° , then m ($\angle$ D) = ...
   (a) 110° (b) 90° (c) 70° (d) 35°
4. The inscribed angle subtended by a minor arc in the circle is ....
   (a) reflex. (b) obtuse. (c) right. (d) acute.
5. If M and N are two circles touching internally , their radii lengths are 4 cm. and 9 cm. , then MN = ... cm.
   (a) 4 (b) 5 (c) 9 (d) 13
6. The measure of the interior angle of the regular hexagon equals ....
   (a) 60° (b) 108° (c) 120° (d) 144°

2 **[a] In the opposite figure :**



$\overline{AB}$ and $\overline{AC}$ are two chords equal in length in the circle M , X is the midpoint of $\overline{AB}$, Y is the midpoint of $\overline{AC}$ and m ($\angle$ CAB) = 70°

1. **Find with proof :** m ($\angle$ DME)
2. **Prove that :** XD = YE

[b] In the opposite figure :

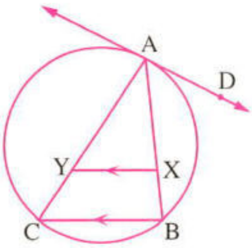
ABC is a triangle inside the circle

, $\overleftrightarrow{AD}$ is a tangent to the circle at A , X ∈ $\overline{AB}$,

Y ∈ $\overline{AC}$ where $\overline{XY} \parallel \overline{BC}$

Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle

passing through the points A , X and Y



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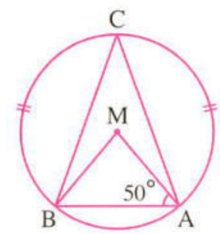
3 **[a] In the opposite figure :**

A circle M , m $(\overset{\frown}{AC})$ = m $(\overset{\frown}{BC})$

and m (∠ MAB) = 50°

Find with proof : 1 m (∠ C)   2 m $(\overset{\frown}{AC})$



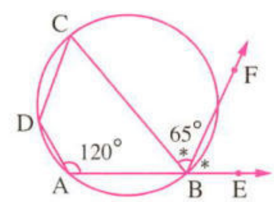
[b] In the opposite figure :

ABCD is a cyclic quadrilateral where

m (∠ A) = 120° , E ∈ $\overrightarrow{AB}$,

$\overrightarrow{BF}$ bisects ∠ EBC and m (∠ FBC) = 65°

Find with proof : 1 m (∠ C)   2 m (∠ D)



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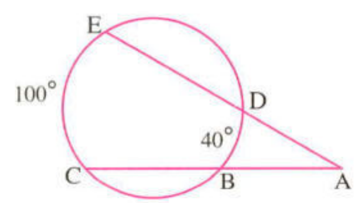
4 **[a] In the opposite figure :**

m $(\overset{\frown}{EC})$ = 100° ,

m $(\overset{\frown}{DB})$ = 40°

Find with proof : m (∠ A)



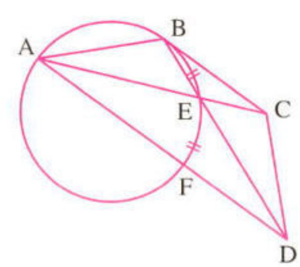
[b] In the opposite figure :

$\overline{BC}$ is a tangent-segment

to the circle at B ,

E is the midpoint of $\overset{\frown}{BF}$

Prove that : ABCD is a cyclic quadrilateral.



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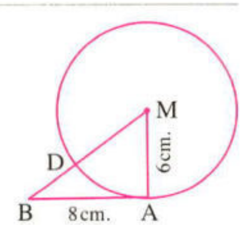
5 **[a] In the opposite figure :**

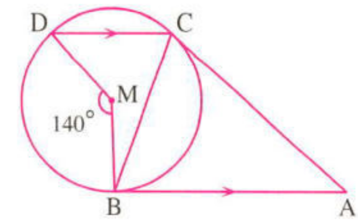
$\overline{AB}$ is a tangent-segment to the circle M at A ,

MA = 6 cm. , AB = 8 cm.

Find with proof : The length of $\overline{DB}$



[b] In the opposite figure :



$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M ,

$\overline{AB} \parallel \overline{CD}$, m (∠ BMD) = 140°

1 **Prove that :** $\overrightarrow{CB}$ bisects ∠ ACD

2 **Find :** m (∠ A)

11 El-Beheira Governorate

Answer the following questions : ***(Calculator is permitted)***

1 Choose the correct answer from the given ones :

1 If the quadrilateral ABCD is a cyclic quadrilateral , then m (∠ A) + m (∠ C) = ...

(a) 90° (b) 120° (c) 180° (d) 360°

2 The inscribed angle opposite to an arc greater than the semicircle is ....

(a) acute. (b) obtuse. (c) straight. (d) right.

3 A chord is 6 cm. long in a circle , the length of its radius is 5 cm. , then it is distant from the centre of the circle by .... cm.

(a) 3 (b) 4 (c) 5 (d) 6

4 A square , its perimeter = 20 cm. , then its area = ... cm^2.

(a) 25 (b) 40 (c) 50 (d) 100

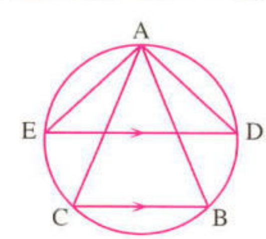
5 If m (∠ A) = 110° , then m (reflex ∠ A) = ...

(a) 70° (b) 110° (c) 180° (d) 250°

6 If ABC is a triangle in which m (∠ B) = m (∠ C) = 60° , then the number of symmetry axes of it equals ....

(a) 3 (b) 2 (c) 1 (d) zero

2 [a] In the opposite figure :



ABC is an inscribed triangle inside a circle ,

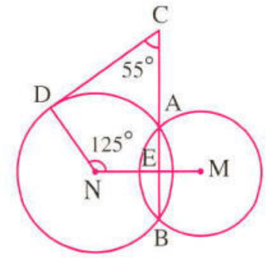
$\overline{DE} \parallel \overline{BC}$

Prove that : m (∠ DAC) = m (∠ EAB)

[b] In the opposite figure :

M and N are two intersecting circles at A and B , $\overline{MN} \cap \overline{AB} = \{E\}$, $C \in \overrightarrow{BA}$, D ∈ the circle N , m (∠ MND) = 125° , m (∠ BCD) = 55°

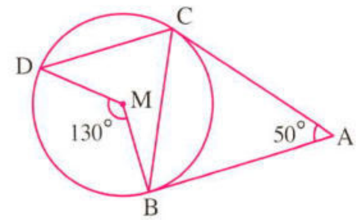
Prove that : $\overleftrightarrow{CD}$ is a tangent to the circle N at D



3 **[a] In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle M , m (∠ A) = 50° , m (∠ BMD) = 130°

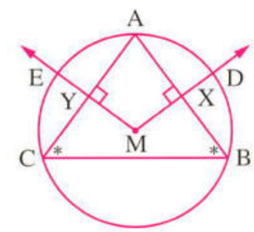
Prove that : $\overline{AB} \parallel \overline{CD}$



[b] In the opposite figure :

ABC is an inscribed triangle inside a circle M , in which m (∠ B) = m (∠ C) , $\overrightarrow{MX} \perp \overline{AB}$ and intersects the circle at D , $\overrightarrow{MY} \perp \overline{AC}$ and intersects the circle at E

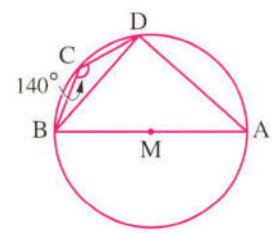
Prove that : XD = YE



4 **[a] In the opposite figure :**

ABCD is a quadrilateral inscribed in the circle M , where $M \in \overline{AB}$, CB = CD , m (∠ BCD) = 140°

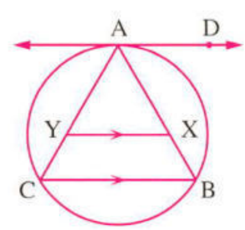
Find : 1 m (∠ A) 2 m (∠ ADC)



[b] In the opposite figure :

ABC is a triangle inscribed in a circle , $\overleftrightarrow{AD}$ is a tangent to the circle at A , $X \in \overline{AB}$, $Y \in \overline{AC}$ where $\overline{XY} \parallel \overline{BC}$

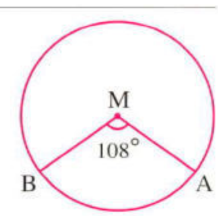
Prove that : $\overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y



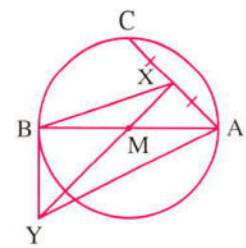
5 **[a] In the opposite figure :**

M is a circle with radius length 5 cm. , m (∠ AMB) = 108°

Find : the length of $\widehat{AB}$ ($\pi = 3.14$)



[b] In the opposite figure :



$\overline{AB}$ is a diameter in the circle M ,

X is the midpoint of $\overline{AC}$,

$\overrightarrow{XM}$ intersects the tangent to the circle at B at Y

Prove that : The figure AXBY is a cyclic quadrilateral.

12 El-Menia Governorate

Answer the following questions :

1 Choose the correct answer from the given ones :

1 The number of the common tangents of two distant circles is ....

(a) 1 (b) 2 (c) 3 (d) 4

2 The number of symmetry axes of any circle is ....

(a) zero (b) 1 (c) 2 (d) an infinite number.

3 The inscribed angle opposite to an arc greater than a semicircle is ....

(a) acute. (b) right. (c) obtuse. (d) reflex.

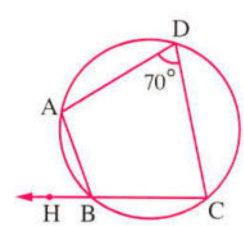
4 In Δ ABC , if $(AB)^2 = (AC)^2 + (BC)^2 + 5$, then $\angle$ B is ....

(a) right. (b) acute. (c) reflex. (d) obtuse.

5 If the area of a triangle is 24 cm^2 and its height is 8 cm. , then the length of the corresponding base is .... cm.

(a) 3 (b) 6 (c) 16 (d) 9

6 **In the opposite figure :**



If m ($\angle$ ADC) = 70° ,

then m ($\angle$ ABH) = ...

(a) 35° (b) 70°

(c) 110° (d) 140°

2 [a] Two circles M and N with radii lengths 9 cm. and 4 cm. respectively.

Show the position of each of them with respect to the other in each of the following cases : 1 MN = 10 cm. 2 MN = 5 cm. 3 MN = 3 cm.

[b] In the opposite figure :

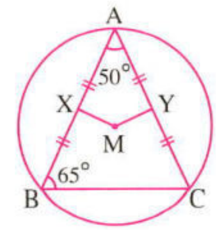
$m(\angle A) = 50^\circ$, $m(\angle B) = 65^\circ$,

X is the midpoint of $\overline{AB}$,

Y is the midpoint of $\overline{AC}$

1 **Calculate :** $m(\angle XMY)$

2 **Prove that :** $MX = MY$

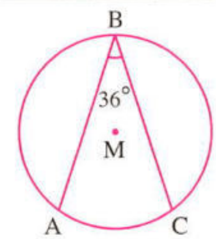


3 **[a] In the opposite figure :**

M is a circle with radius length 5 cm. ,

$m(\angle B) = 36^\circ$

Find : The length of $\overset{\frown}{AC}$ ($\pi = 3.14$)



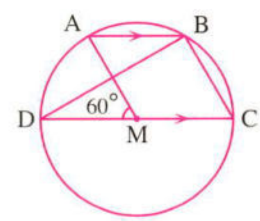
[b] In the opposite figure :

$\overline{DC}$ is a diameter in the circle M ,

$\overline{DC} \parallel \overline{AB}$,

$m(\angle AMD) = 60^\circ$

Find with proof : $m(\angle ABD)$ and $m(\angle BCD)$



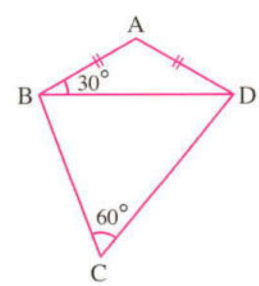
4 **[a]** State three cases of the quadrilateral to be cyclic.

[b] In the opposite figure :

$AB = AD$,

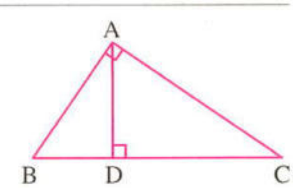
$m(\angle ABD) = 30^\circ$ and $m(\angle BCD) = 60^\circ$

Prove that : The figure ABCD is a cyclic quadrilateral.



5 **[a] In the opposite figure :**

Prove that : $\overleftrightarrow{AB}$ is a tangent to the circle passing through the points A , C and D

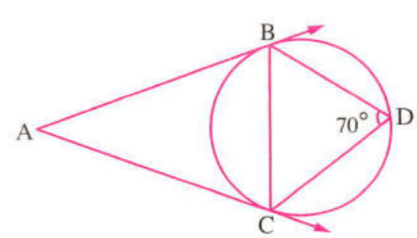


[b] In the opposite figure :

$\overrightarrow{AB}$ and $\overrightarrow{AC}$ are two tangents to the circle at B and C ,

$m(\angle CDB) = 70^\circ$

Find with proof : $m(\angle CAB)$



13 Assiut Governorate

Answer the following questions : ***(Calculator is allowed)***

1 Choose the correct answer :

1. The number of circles that can be drawn passing through both ends of the line segment $\overline{AB}$ equals ....
   (a) zero (b) 1 (c) 2 (d) an infinite number.

2. If the lengths of two adjacent sides of a parallelogram are 3 cm. , 5 cm. , then the perimeter of this parallelogram is .... cm.
   (a) 8 (b) 15 (c) 16 (d) 18

3. If ABCD is a cyclic quadrilateral , then m (∠ A) + m (∠ C) = ...
   (a) 180° (b) 90° (c) 270° (d) 360°

4. ABC is a triangle where $(BC)^2 = (AB)^2 + (AC)^2$, m (∠ B) = 40° , then m (∠ C) = ...
   (a) 40° (b) 50° (c) 90° (d) 140°

5. The number of common tangents of two circles which are one circle inside the other without touching equals ....
   (a) 3 (b) 2 (c) 1 (d) zero.

6. If the two similar triangles are congruent , then the enlargement ratio equals ....
   (a) 2 (b) 1 (c) zero (d) $\frac{3}{2}$

2 [a] In the opposite figure :

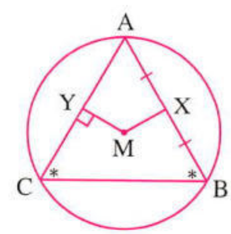
The triangle ABC is inscribed in the circle M ,

, m (∠ B) = m (∠ C) ,

X is the midpoint of $\overline{AB}$,

$\overline{MY} \perp \overline{AC}$

Prove that : MX = MY



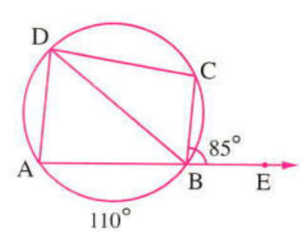
[b] In the opposite figure :

E ∈ $\overrightarrow{AB}$, E ∉ $\overline{AB}$,

m ($\widehat{AB}$) = 110° ,

m (∠ CBE) = 85°

Find with proof : m (∠ BDC)

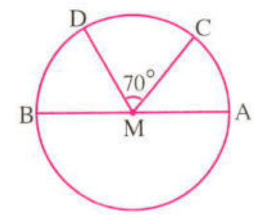


3 [a] **In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M , m ($\angle$ CMD) = 70° ,

m ($\overset{\frown}{AC}$) : m ($\overset{\frown}{DB}$) = 5 : 6

Find : m ($\overset{\frown}{AC}$) showing the steps.



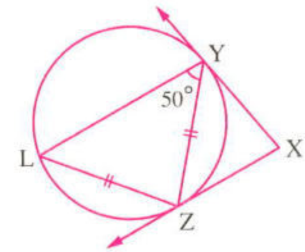
[b] **In the opposite figure :**

$\overrightarrow{XY}$, $\overrightarrow{XZ}$ are two tangents to the circle at Y , Z

, YZ = LZ ,

m ($\angle$ ZYL) = 50° ,

Find with proof : m ($\angle$ X)



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4 [a] If M is a circle with radius length 13 cm. , $\overleftrightarrow{MA} \perp$ L , where A $\in$ L , **Determine the position of the straight line L with respect to the circle M in each of the following cases :** 1 MA = 13 cm. 2 MA = 10 cm. 3 MA = 15 cm.

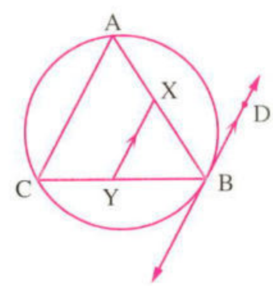
[b] **In the opposite figure :**

ABC is a triangle inscribed in a circle ,

$\overleftrightarrow{BD}$ is a tangent to the circle at B , X $\in \overline{AB}$,

Y $\in \overline{BC}$, where $\overline{XY}$ // $\overleftrightarrow{BD}$

Prove that : AXYC is a cyclic quadrilateral.



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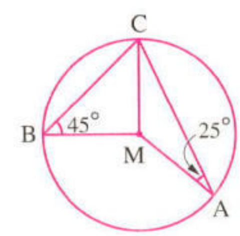
5 [a] **In the opposite figure :**

A circle of centre M , m ($\angle$ MAC) = 25° ,

m ($\angle$ MBC) = 45°

Find with proof : m ($\angle$ AMB)



[b] **In the opposite figure :**

ABCD is a quadrilateral inscribed in a circle ,

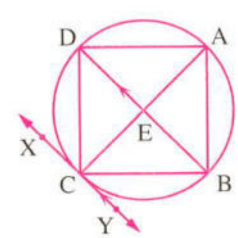
its two diagonals intersect at E ,

$\overleftrightarrow{XY}$ is drawn to be a tangent to the circle

at C where $\overleftrightarrow{XY}$ // $\overline{BD}$

Prove that : 1 Δ BDC is an isosceles triangle.

2 $\overleftrightarrow{BC}$ touches the circle passing through the vertices of Δ ABE



14 Qena Governorate

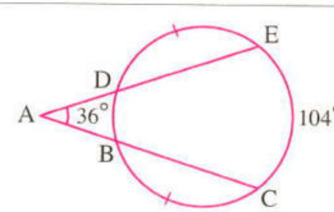
محافظة قنا

Answer the following questions : *(Calculator is permitted)*

1 Choose the correct answer from those given :

1. If AB = 8 cm. , the circumference of the smallest circle passing through the two points A and B equals …………… cm.
   (a) 8π (b) 12π (c) 16π (d) 64π
2. The sum of measures of two supplementary angles equals ……………
   (a) 90° (b) 180° (c) 270° (d) 360°
3. ABCD is a square drawn inside a circle , then m $(\overset{\frown}{AB})$ = ……………
   (a) 60° (b) 90° (c) 120° (d) 180°
4. In Δ ABC , if $(AC)^2 + (BC)^2 = (AB)^2 + 5$, then $\angle$ C is ……………
   (a) obtuse. (b) right. (c) acute. (d) straight.
5. In the cyclic quadrilateral ABCD if m ($\angle$ A) = 3 m ($\angle$ C) , then m ($\angle$ A) = ……………
   (a) 45° (b) 90° (c) 120° (d) 135°
6. The width of a rectangle is 5 cm. and its diagonal length is 13 cm. , then its length is …………… cm.
   (a) 5 (b) 6 (c) 12 (d) 10

2 [a] In the opposite figure :

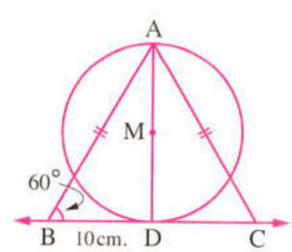


m ($\angle$ A) = 36° , m $(\overset{\frown}{EC})$ = 104° ,

m $(\overset{\frown}{BC})$ = m $(\overset{\frown}{DE})$

Find : m $(\overset{\frown}{BD})$, m $(\overset{\frown}{DE})$

[b] In the opposite figure :



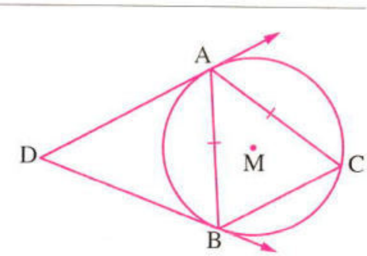
$\overleftrightarrow{CB}$ is a tangent to the circle M ,

AB = AC , BD = 10 cm. ,

m ($\angle$ CBA) = 60°

Find with proof : The perimeter of Δ ABC

3 [a] In the opposite figure :



If $\overrightarrow{DA}$, $\overrightarrow{DB}$ are two tangents to the circle M , AB = AC

, **prove that :** $\overleftrightarrow{AC}$ is a tangent to the circle passing through the vertices of Δ ABD

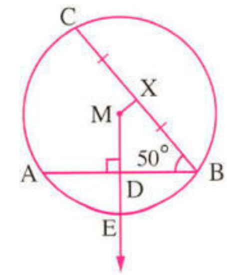
[b] In the opposite figure :

$\overline{AB}$, $\overline{BC}$ are two chrods in the circle M whose radius length is 5 cm. , $\overrightarrow{MD} \perp \overline{AB}$ intersecting the circle M at E ,

X is the midpoint of $\overline{BC}$, AB = 8 cm. ,

m (∠ ABC) = 50°

Find with proof : 1 m (∠ DMX) 2 The length of $\overline{DE}$

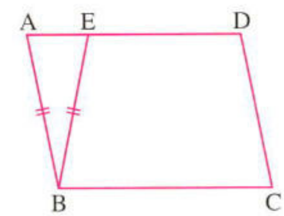


4 **[a] In the opposite figure :**

ABCD is a parallelogram ,

AB = BE

Prove that : BEDC is a cyclic quadrilateral.



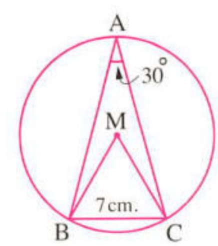
[b] In the opposite figure :

m (∠ A) = 30° ,

BC = 7 cm.

Find : the area of the circle M

$\left(\text{where } \pi = \frac{22}{7}\right)$

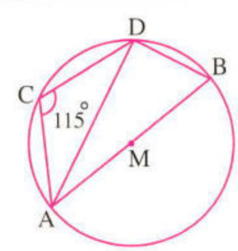


5 **[a] In the opposite figure :**

$\overline{AB}$ is a diameter in the circle M ,

m (∠ ACD) = 115°

Find with proof : m (∠ DAB)



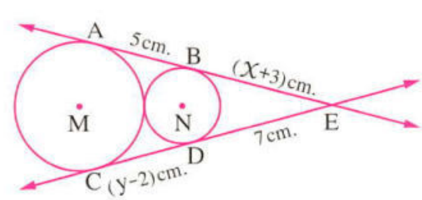
[b] In the opposite figure :

$\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$ are common tangents to the two circles M , N , $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \{E\}$,

ED = 7 cm. , AB = 5 cm. , EB = $(X + 3)$ cm. ,

CD = (y – 2) cm.

Find with proof : The values of X , y



15 Matrouh Governorate

Answer the following questions : ***(Calculator is allowed)***

1 **Choose the correct answer from the given ones :**

1 The inscribed angle in a semicircle is .... angle.

(a) an acute (b) an obtuse (c) a right (d) a straight

2 The sum of the measures of the interior angles of a quadrilateral equals ....

(a) 180° (b) 360° (c) 270° (d) 540°

3 A circle has a circumference of 8π length unit, so the length of the diameter = ... length unit.

(a) 8 (b) 4 (c) π (d) 2π

4 If the two circles M , N are touching externally , the radius length of one of them is 5 cm. , and MN = 9 cm. , then the radius length of the other circle equals .... cm.

(a) 3 (b) 4 (c) 7 (d) 14

5 If a rectangle has a length of 3 cm. and a width of 2 cm. , then its surface area = ... cm^2.

(a) 4 (b) 5 (c) 6 (d) 10

6 XYZL is a cyclic quadrilateral in which m (∠ X) = 2 m (∠ Z) , then m (∠ Z) = ...

(a) 30° (b) 60° (c) 90° (d) 120°

---

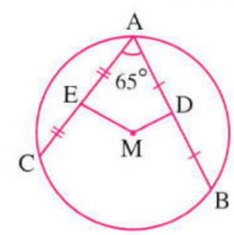
2 [a] **In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two chords in a circle M ,

D , E are the two midpoints of $\overline{AB}$, $\overline{AC}$,

m (∠ BAC) = 65°

Find with proof : m (∠ DME)

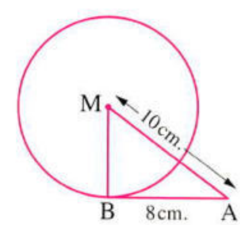


[b] **In the opposite figure :**

$\overline{AB}$ is a tangent-segment to the circle M at B ,

AB = 8 cm. , MA = 10 cm.

Find : The surface area of the triangle ABM



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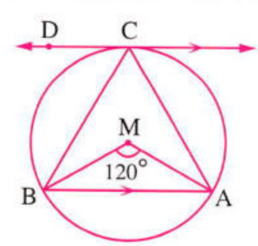
3 [a] **In the opposite figure :**

$\overleftrightarrow{CD}$ is a tangent to the circle at C ,

$\overleftrightarrow{CD}$ // $\overline{AB}$,

m (∠ AMB) = 120°

Prove that : The triangle CAB is an equilateral triangle.



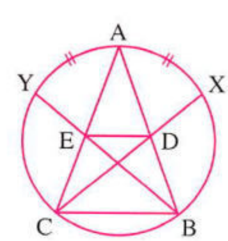
[b] **In the opposite figure :**

ABC is a triangle inscribed in a circle ,

$X \in \widehat{AB}$, $Y \in \widehat{AC}$, where m $(\widehat{AX})$ = m $(\widehat{AY})$,

$\overline{CX} \cap \overline{AB} = \{D\}$, $\overline{BY} \cap \overline{AC} = \{E\}$

Prove that : BCED is a cyclic quadrilateral.

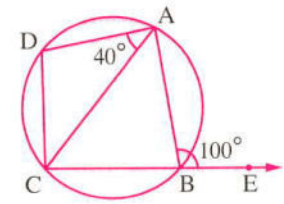


4 [a] **In the opposite figure :**

m (∠ ABE) = 100° ,

m (∠ CAD) = 40° , E ∈ $\overrightarrow{CB}$

Prove that : The triangle ADC is an isosceles triangle.



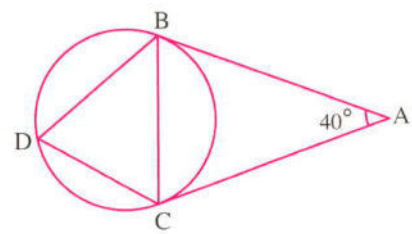
[b] **In the opposite figure :**

$\overline{AB}$ and $\overline{AC}$ are two tangent-segments

to the circle at B , C ,

m (∠ A) = 40°

Find with proof : m (∠ D)

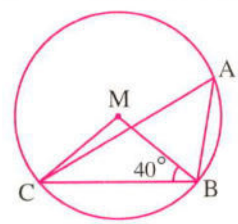


5 [a] **In the opposite figure :**

A circle with centre M ,

m (∠ MBC) = 40°

Find with proof : m (∠ BAC)



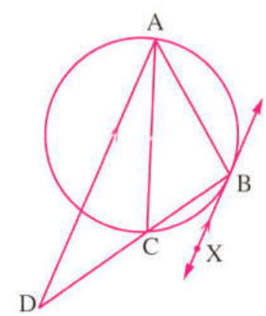
[b] **In the opposite figure :**

ABC is an inscribed triangle inside a circle ,

$\overleftrightarrow{BX}$ is a tangent to the circle at B ,

$\overline{AD}$ // $\overleftrightarrow{BX}$

Prove that : $\overline{AB}$ is a tangent-segment to the circle passing through the vertices of the triangle ACD



Exam 1 Port Said Governorate

First Multiple choice questions

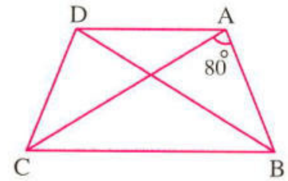
Choose the correct answer from those given :

1 **In the opposite figure :**

ABCD is a cyclic quadrilateral

in which m (∠ BAC) = 80°

, then m (∠ BDC) = ...



(a) 40° (b) 80° (c) 90° (d) 100°

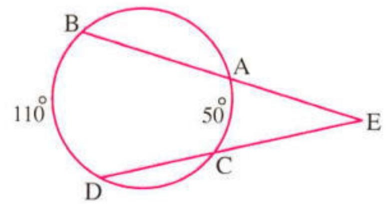
2 The chord which passes through the centre of the circle is called ....

(a) tangent. (b) secant. (c) diameter. (d) radius.

3 **In the opposite figure :**

m $(\widehat{AC})$ = 50° , m $(\widehat{BD})$ = 110°

, then m (∠ E) = ...



(a) 10° (b) 20°

(c) 30° (d) 40°

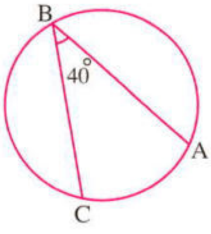
4 If the point A ∈ circle M which its diameter of length 6 cm. , then MA = ... cm.

(a) 3 (b) 4 (c) 5 (d) 6

5 **In the opposite figure :**

m (∠ ABC) = 40°

, then m $(\widehat{AC})$ = ...



(a) 20° (b) 40°

(c) 60° (d) 80°

6 The supplementary for the angle whose measure 70° equals ....

(a) 20° (b) 70° (c) 110° (d) 180°

7 The two tangents drawn from the endpoints of a diameter of circle are ....

(a) equal in length. (b) parallel. (c) intersection. (d) perpendicular.

8 The number of circles that can be drawn passing through the endpoints of a line segment is ....

(a) one. (b) two.

(c) three. (d) an infinite number.

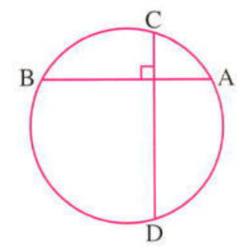
---

9 A rectangle of length 5 cm. and width 3 cm. , then its area = ... cm^2.

(a) 4 (b) 8 (c) 15 (d) 16

---

10 **In the opposite figure :**



$\overline{AB} \perp \overline{CD}$

, then m $(\overset{\frown}{AC})$ + m $(\overset{\frown}{BD})$ = ...

(a) 45° (b) 90°

(c) 180° (d) 270°

---

11 We can draw a circle passes through the vertices of ....

(a) trapezium. (b) parallelogram. (c) rhombus. (d) rectangle.

---

12 The number of common tangents for two circles touching internally equals ....

(a) zero. (b) one. (c) two. (d) three.

---

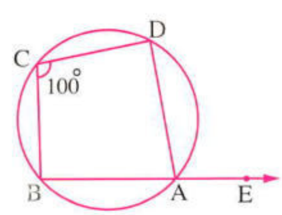
13 If M and N are two centres of two circles touching externally whose radii are 5 cm. , 9 cm.

, then MN = ... cm.

(a) 14 (b) 4 (c) 5 (d) 9

---

14 **In the opposite figure :**



m (∠ C) = 100°

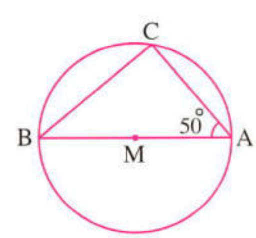
, then m (∠ DAE) = ...

(a) 80° (b) 100°

(c) 50° (d) 90°

---

15 **In the opposite figure :**



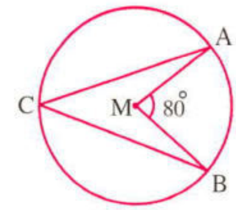
$\overline{AB}$ is a diameter in the circle M , m (∠ A) = 50°

, then m (∠ ABC) = ...

(a) 25° (b) 40°

(c) 50° (d) 30°

16 **In the opposite figure :**



m (∠ AMB) = 80°

, then m (∠ ACB) = ...

(a) 40°
(b) 80°
(c) 60°
(d) 20°

---

17 The number of axes of symmetry of the equilateral triangle is ....

(a) zero.
(b) one.
(c) two.
(d) three.

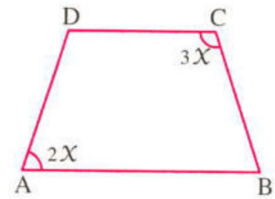
---

18 The measure of the arc which is quarter circle is ....

(a) 60°
(b) 90°
(c) 120°
(d) 180°

---

19 **In the opposite figure :**



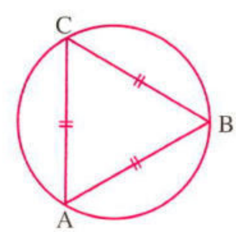
ABCD is a cyclic quadrilateral in which m (∠ A) = 2 X

, m (∠ C) = 3 X , then X = ...

(a) 20°
(b) 30°
(c) 32°
(d) 36°

---

20 **In the opposite figure :**



Δ ABC is an equilateral triangle

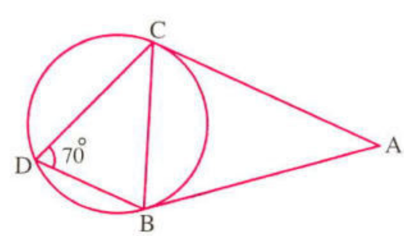
, the length of $\overset{\frown}{AB}$ = 8 cm.

, then the circumference of its circumcircle = ... cm.

(a) 24
(b) 48
(c) 16
(d) 40

---

21 **In the opposite figure :**



$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle from A , m (∠ D) = 70°

, then m (∠ A) = ...

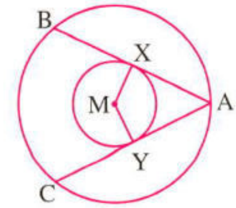
(a) 35°
(b) 70°
(c) 40°
(d) 20°

Second Essay questions

22 **In the opposite figure :**

$\overline{AB}$, $\overline{AC}$ are two tangent-segments to the smaller circle M

Show that : AB = AC



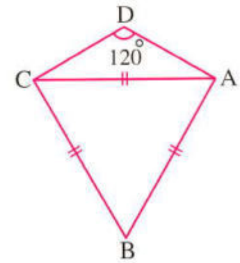
23 **In the opposite figure :**

m (∠ D) = 120°

, Δ ABC is an equilateral triangle

Show that :

ABCD is cyclic quadrilateral

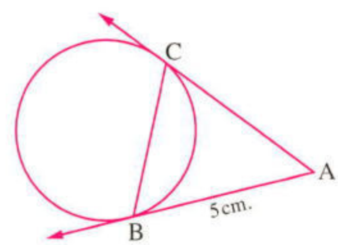


24 **In the opposite figure :**

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents

, m $(\overset{\frown}{BC})$ = 120° , AB = 5 cm.

Find the perimeter of Δ ABC



Exam 2

First Multiple choice questions

Choose the correct answer from those given :

1 A chord of length 8 cm. , is drawn in a circle of diameter length 10 cm. , then its distance from the centre is .... cm.

(a) 2 (b) 4 (c) 3 (d) 6

2 The length of the side opposite to the angle of measure 30° in the right-angled triangle equals .... the length of the hypotenuse.

(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{2}$ (d) 2

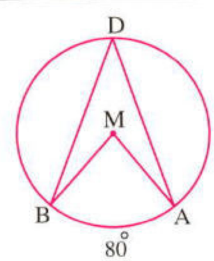
3 **In the opposite figure :**

A circle of centre M if m $(\overset{\frown}{AB})$ = 80°

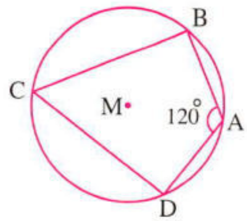
, then m (∠ ADB) = ...

(a) 40° (b) 60°

(c) 120° (d) 160°



4 **In the opposite figure :**



M is a circle , m (∠ A) = 120°

, then m (∠ C) = ...

(a) 110° (b) 60°

(c) 55° (d) 180°

5 If the straight line L is a tangent to the circle M of diameter length 10 cm. , then the distance between L and the centre of the circle equal .... cm.

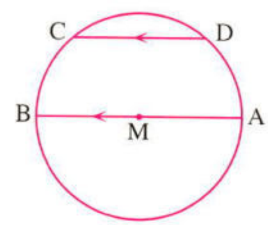
(a) 3 (b) 4 (c) 6 (d) 8

6 If two chords intersect at a point inside a circle , then the measure of the included angle equals .... of the two measures of the two opposite arcs.

(a) half the difference. (b) half the sum.

(c) twice the sum. (d) twice the difference.

7 **In the opposite figure :**



M is a circle , m $(\overset{\frown}{BC}) = 50°$, $\overline{DC} \parallel \overline{AB}$

, then m $(\overset{\frown}{DC})$ = ...

(a) 100° (b) 60°

(c) 120° (d) 80°

8 M and N are two circles whose radii lengths are 5 cm. , 3 cm. respectively , if MN = 8 cm. , then the two circles are ....

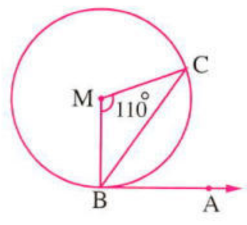
(a) touching internally. (b) intersecting.

(c) touching externally. (d) distant.

9 If $\tan(X + 10°) = \sqrt{3}$ where X is the measure of an acute angle , then X = ...

(a) 30° (b) 45° (c) 50° (d) 60°

10 **In the opposite figure :**



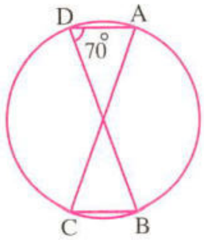
$\overrightarrow{BA}$ is a tangent to the circle M at B , m (∠ CMB) = 110

, then m (∠ ABC) = ...°

(a) 55° (b) 35°

(c) 95° (d) 110°

11 **In the opposite figure :**



If m (∠ ADB) = 70°

, then m (∠ ACB) = ...

(a) 35° (b) 70°

(c) 90° (d) 140°

---

12 The number of circles that pass through three collinear points equals ....

(a) zero. (b) one.

(c) three. (d) an infinite number.

---

13 The two diagonals are perpendicular and not equal in length in the ....

(a) rhombus. (b) trapezium. (c) square. (d) rectangle.

---

14 ABCD is a cyclic quadrilateral in which m (∠ A) = 2 m (∠ C) , then m (∠ A) = ...

(a) 30° (b) 60° (c) 90° (d) 120°

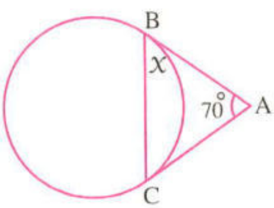
---

15 The measure of the arc which represents $\frac{1}{6}$ the circumference of the circle equals ....

(a) 60° (b) 90° (c) 120° (d) 300°

---

16 **In the opposite figure :**



If $\overline{AB}$ and $\overline{AC}$ are two tangent-segments to the circle

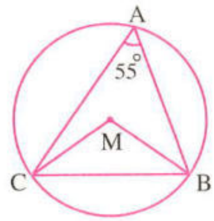
, m (∠ A) = 70° , then X = ...

(a) 55° (b) 50°

(c) 60° (d) 70°

---

17 **In the opposite figure :**



If m (∠ A) = 55°

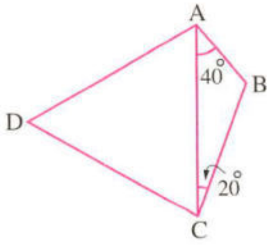
, then m (∠ BMC) = ...

(a) 110° (b) 55°

(c) 35° (d) 25°

---

18 **In the opposite figure :**



ABCD is a cyclic quadrilateral , m (∠ BAC) = 40°

, m (∠ BCA) = 20° , then m (∠ D) = ...

(a) 20° (b) 40°

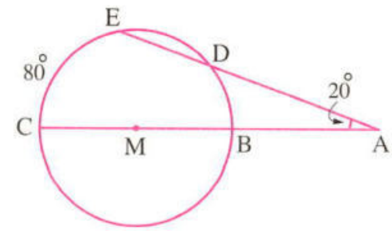
(c) 60° (d) 120°

19 The number of common tangents of two circles touching externally is ....

(a) zero (b) 1 (c) 2 (d) 3

---

20 **In the opposite figure :**



If $\overline{BC}$ is a diameter of the circle M

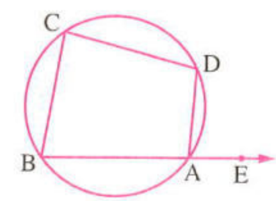
, m (∠ A) = 20° , m $(\overset{\frown}{CE})$ = 80°

, then m $(\overset{\frown}{DE})$ = ...

(a) 40° (b) 120° (c) 60° (d) 100°

---

21 **In the opposite figure :**



ABCD is a quadrilateral drawn inside the circle , E ∈ $\overrightarrow{BA}$

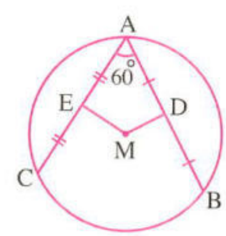
, then m (∠ DAE) = m (∠ ....)

(a) B (b) C

(c) D (d) DAB

Second Essay questions

22 **In the opposite figure :**



$\overline{AB}$, $\overline{AC}$ are two chords of the circle M

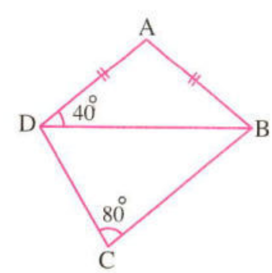
, D is the midpoint of $\overline{AB}$, E is the midpoint of $\overline{AC}$

, m (∠ BAC) = 60°

Find with proof : m (∠ DME)

---

23 **In the opposite figure :**



AB = AD

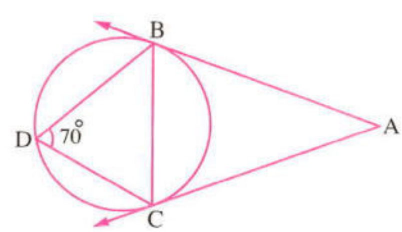
, m (∠ ADB) = 40° , m (∠ BCD) = 80°

Prove that :

ABCD is a cyclic quadrilateral.

---

24 **In the opposite figure :**



$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle at B , C

, m (∠ BDC) = 70°

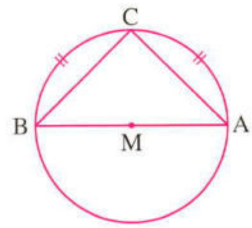
Find m (∠ A)

Exam 3

First Multiple choice questions

Choose the correct answer from those given :

1 **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M

, m $(\overset{\frown}{AC})$ = m $(\overset{\frown}{BC})$, then m (∠ CBA) = ……………

(a) 40° (b) 45°

(c) 50° (d) 90°

---

2 A circle of circumference 8 π cm. , then its diameter length equals …………… cm.

(a) 2 (b) 4 (c) 8 (d) 16

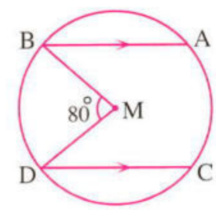
---

3 A triangle ABC in which $(AB)^2 > (BC)^2 + (AC)^2$, then ∠ C is ……………

(a) an acute. (b) a right. (c) an obtuse. (d) a straight.

---

4 **In the opposite figure :**



A circle M , $\overline{AB}$ // $\overline{CD}$, m (∠ BMD) = 80°

, then m $(\overset{\frown}{AC})$ = ……………

(a) 20° (b) 40°

(c) 80° (d) 160°

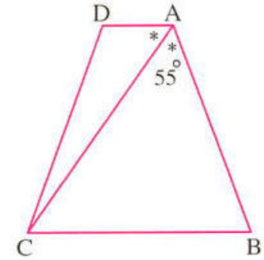
---

5 If the two circles M and N have four common tangents and the radius length of the circle M = 3 cm. , the radius length of the circle N = 5 cm. , then ……………

(a) MN = 2 cm. (b) MN = 8 cm. (c) MN < 8 cm. (d) MN > 8 cm.

---

6 **In the opposite figure :**



ABCD is a cyclic quadrilateral

, $\overrightarrow{AC}$ bisects ∠ BAD , m (∠ BAC) = 55°

, then m (∠ BCD) = ……………

(a) 55° (b) 70°

(c) 110° (d) 125°

7 The number of axes of symmetry of equilateral triangle equal ....

(a) zero (b) 1 (c) 2 (d) 3

8 The ratio between the measure of the central angle and the inscribed angle subtended by the same arc is ....

(a) 3 : 1 (b) 2 : 1 (c) 1 : 2 (d) 1 : 3

9 If M is a circle of radius length 5 cm. , A is a point outside the circle , then MA may be equal .... cm.

(a) 3 (b) 5 (c) 8 (d) 4

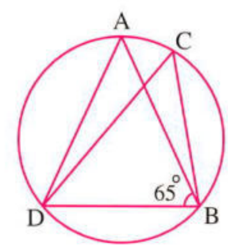
10 **In the opposite figure :**

If m (∠ ABD) = 65° , AB = AD

, then m (∠ BCD) = ...

(a) 15° (b) 25°

(c) 30° (d) 50°



11 **In the opposite figure :**

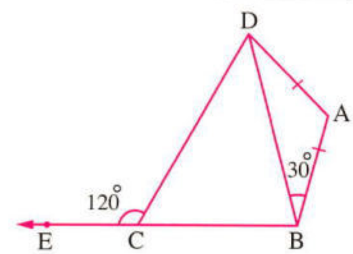
ABCD is a quadrilateral , m (∠ ABD) = 30°

, m (∠ DCE) = 120° , AB = AD

, then the figure ABCD is ....

(a) rectangle. (b) rhombus.

(c) square. (d) cyclic quadrilateral.



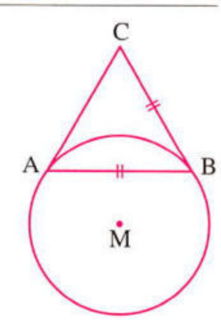
12 **In the opposite figure :**

$\overline{CB}$, $\overline{CA}$ are two tangent-segments to the circle M

, CB = BA , then m (∠ C) = ...

(a) 60° (b) 120°

(c) 90° (d) 100°



13 M and N are two circles , their radii lengths are 4 cm. and 3 cm. , if MN = 9 cm. , then the two circles are ....

(a) touching. (b) concentric. (c) distant. (d) intersecting.

14 .... is a rhombus with one right angle.

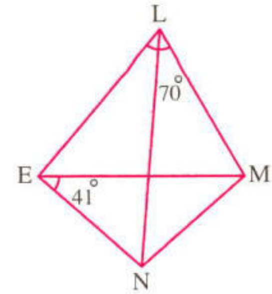
(a) The rectangle. (b) The square. (c) The parallelogram. (d) The trapezium.

15 ABC is an equilateral triangle drawn inside a circle , then m $(\overset{\frown}{AB})$ = ...

(a) 30°   (b) 60°   (c) 90°   (d) 120°

---

16 **In the opposite figure :**



LMNE is a cyclic quadrilateral

, m (∠ MLE) = 70° , m (∠ MEN) = 41°

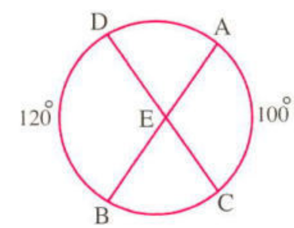
, then m (∠ EMN) = ...

(a) 70°   (b) 41°

(c) 29°   (d) 110°

---

17 **In the opposite figure :**



If m $(\overset{\frown}{AC})$ = 100° , m $(\overset{\frown}{DB})$ = 120°

, then m (∠ AED) = ...

(a) 110°   (b) 55°

(c) 70°   (d) 100°

---

18 It is impossible to draw a circle passing through the vertices of a ....

(a) triangle.   (b) square.   (c) rhombus.   (d) rectangle.

---

19 The angle of tangency is the included angle between ....

(a) two chords.   (b) two tangents.

(c) a chord and a tangent.   (d) a chord and a diameter.

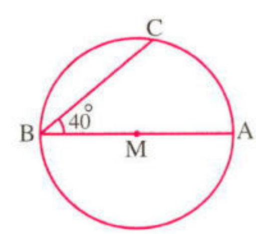
---

20 The figure ABCD is a cyclic quadrilateral if m (∠ A) + m (∠ C) – 100° = ...

(a) 80°   (b) 100°   (c) 180°   (d) 90°

---

21 **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M

, m (∠ ABC) = 40°

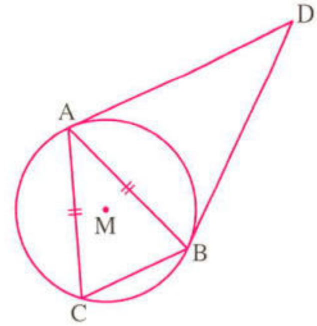
, then m $(\overset{\frown}{BC})$ = ...

(a) 40°   (b) 50°

(c) 80°   (d) 100°

Second | Essay questions

22 **In the opposite figure :**

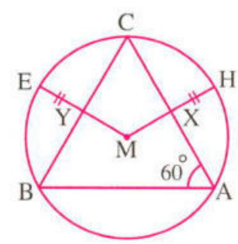


$\overline{AD}$, $\overline{BD}$ are two tangent-segments to the circle M at A , B , C $\in$ the circle M where AB = AC

Prove that :

$\overleftrightarrow{AC}$ is a tangent to the circle passing through the vertices of Δ ABD

23 **In the opposite figure :**



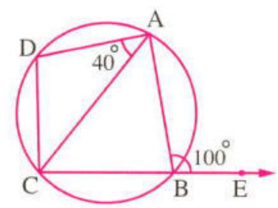
M is a circle , m ($\angle$ A) = 60°

, X is the midpoint of $\overline{AC}$

, Y is the midpoint of $\overline{BC}$, HX = EY

Prove that : Δ ABC is an equilateral triangle.

24 **In the opposite figure :**



m ($\angle$ ABE) = 100°

, m ($\angle$ CAD) = 40°

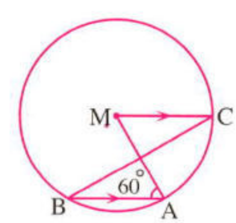
Prove that : m ($\overset{\frown}{CD}$) = m ($\overset{\frown}{AD}$)

Exam 4

First | Multiple choice questions

Choose the correct answer from those given :

1 **In the opposite figure :**



$\overline{AB}$ is a chord in the circle M

, $\overline{AB}$ // $\overline{MC}$, m ($\angle$ MAB) = 60°

, then m ($\angle$ ABC) = ...

(a) 30° (b) 60° (c) 90° (d) 120°

2 The point of concurrence of the medians of the triangle divides each of them in the ratio of .... from the base.

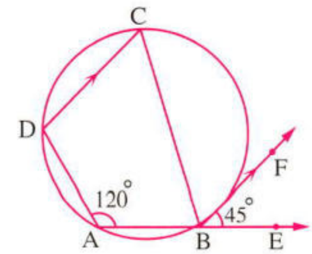
(a) 3 : 9 (b) 3 : 1 (c) 4 : 2 (d) 2 : 4

3 The area of the circle is 25 π cm^2 , the straight line L is of distance 5 cm. of its centre , then L is ....

(a) outside the circle. (b) a tangent to the circle.

(c) a secant of the circle. (d) passing through the centre.

4 **In the opposite figure :**



$\overrightarrow{BF} \parallel \overline{DC}$, m (∠ DAB) = 120°

, m (∠ FBE) = 45°

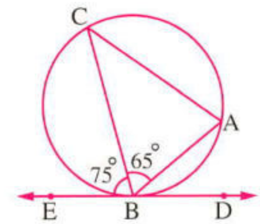
, then m (∠ ADC) = ...

(a) 75° (b) 60°

(c) 105° (d) 120°

---

5 **In the opposite figure :**



$\overleftrightarrow{DE}$ is a tangent to the circle at B

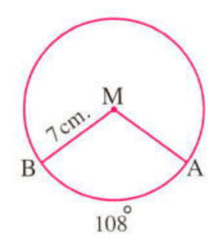
, m (∠ ABC) = 65° , m (∠ CBE) = 75°

, then m (∠ C) = ...

(a) 20° (b) 40° (c) 50° (d) 80°

---

6 **In the opposite figure :**



M is a circle its radius length is 7 cm.

, m ($\overset{\frown}{AB}$) = 108°

, then the length of $\overset{\frown}{AB}$ = ... cm. $\left(\pi = \frac{22}{7}\right)$

(a) 14 (b) 13.2

(c) 7 (d) 6.6

---

7 If AB = 4 cm. , then the radius length of the smallest circle which passes through the two points A and B equals .... cm.

(a) 2 (b) 3 (c) 4 (d) 5

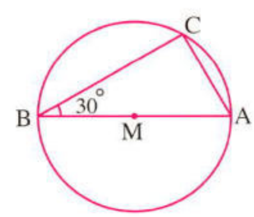
---

8 Measuring the angle of the vertex of a regular hexagon = ...

(a) 60° (b) 108° (c) 120° (d) 135°

---

9 **In the opposite figure :**



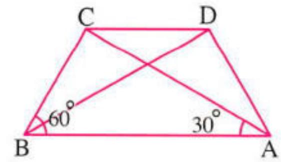
$\overline{AB}$ is a diameter in the circle M

, m (∠ B) = 30° , AC = 6 cm.

, then AB = ... cm.

(a) 3 (b) 6 (c) 9 (d) 12

10 **In the opposite figure :**



ABCD is a cyclic quadrilateral

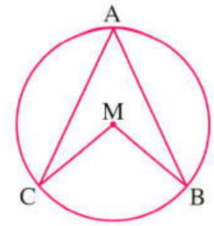
, m (∠ BAC) = 30° , m (∠ ABC) = 60°

, then m (∠ ADB) = ...

(a) 50° (b) 60° (c) 80° (d) 90°

---

11 **In the opposite figure :**



M is a circle , if m (∠ M) – m (∠ A) = 50°

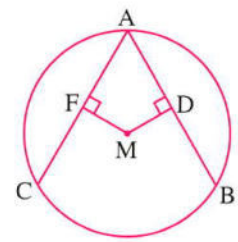
, then m (∠ A) = ...

(a) 40° (b) 50°

(c) 100° (d) 130°

---

12 **In the opposite figure :**



AB = AC , $\overline{MD} \perp \overline{AB}$, $\overline{MF} \perp \overline{AC}$, MD = 6 cm.

, then MF = ... cm.

(a) 12 (b) 8

(c) 6 (d) 3

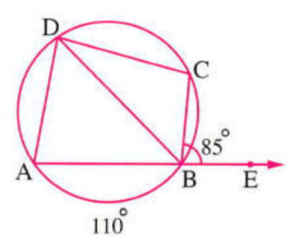
---

13 A square with area 25 cm^2 , its perimeter = ... cm.

(a) 5 (b) 10 (c) 15 (d) 20

---

14 **In the opposite figure :**



If E $\in \overrightarrow{AB}$, m (∠ EBC) = 85° , m ($\widehat{AB}$) = 110°

, then m (∠ BDC) = ...

(a) 30° (b) 55°

(c) 85° (d) 110°

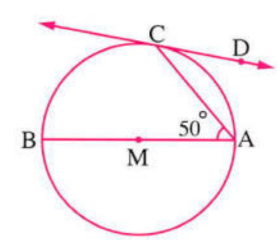
---

15 The inscribed angle which is opposite to the minor arc in a circle is ....

(a) reflex. (b) right. (c) obtuse. (d) acute.

---

16 **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M

, $\overleftrightarrow{DC}$ is a tangent to it at C , m (∠ A) = 50°

, then m (∠ ACD) = ...

(a) 50° (b) 40°

(c) 100° (d) 80°

17 The axis of symmetry of the circle is ....

(a) the diameter.

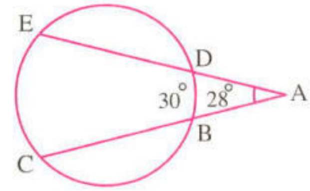
(b) the chord.

(c) the straight line passing through the centre.

(d) the tangent.

---

18 **In the opposite figure :**



m (∠ A) = 28° , m ($\widehat{BD}$) = 30°

, then m ($\widehat{CE}$) = ...

(a) 60° (b) 58°

(c) 86° (d) 108°

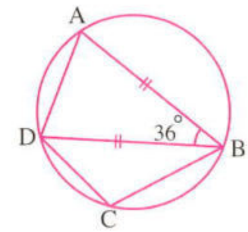
---

19 The number of common tangents of two distant circles is ....

(a) zero (b) 1 (c) 2 (d) 4

---

20 **In the opposite figure :**



If AB = BD , m (∠ ABD) = 36°

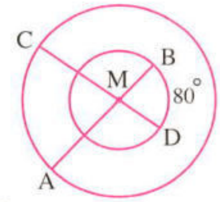
, then m (∠ C) = ...

(a) 140° (b) 70°

(c) 54° (d) 108°

---

21 **In the opposite figure :**



Two concentric circles at M , $\overline{AB} \cap \overline{CD} = \{M\}$

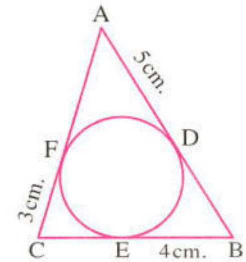
, then m ($\widehat{BD}$) = 80°

, then m ($\widehat{AC}$) = ...

(a) 40° (b) 60° (c) 80° (d) 160°

Second Essay questions

22 **In the opposite figure :**



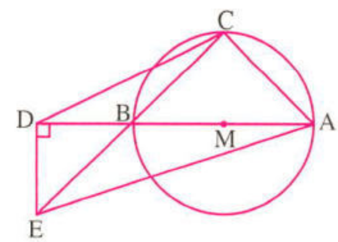
A circle is drawn touching the sides of the triangle ABC

, $\overline{AB}$, $\overline{BC}$, $\overline{AC}$ at D , E , F

, AD = 5 cm. , BE = 4 cm. , CF = 3 cm.

Find the perimeter : of Δ ABC

23 **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M

, $D \in \overrightarrow{AB}$, $D \notin \overline{AB}$

, draw $\overrightarrow{DE} \perp \overrightarrow{AB}$, $C \in \widehat{AB}$, $\overrightarrow{CB} \cap \overrightarrow{DE} = \{E\}$

Prove that : ACDE is a cyclic quadrilateral.

---

24 $\overline{AB}$ is a line segment of length 6 cm. draw a circle passes through the two points A , B with radius length 4 cm. How many circles you can draw ? (Don't remove the ares)

Exam 5

First Multiple choice questions

Choose the correct answer from those given :

1 If the measure of the tangency angle equal 70° , then the measure of the central angle subtended by the same arc equals ....

(a) 35° (b) 70° (c) 140° (d) 105°

---

2 If ABCD is a cyclic quadrilateral , if m (∠ B) = $\frac{1}{4}$ m (∠ D) , then m (∠ B) = ...

(a) 36° (b) 45° (c) 72° (d) 144°

---

3 The number of axes of symmetry of the semicircle is ....

(a) zero (b) 1 (c) 3 (d) an infinite number.

---

4 The length of the arc which represents $\frac{1}{4}$ the circumference of the circle equals ....

(a) $2\pi r$ (b) πr (c) $\frac{1}{2}\pi r$ (d) $4\pi r$

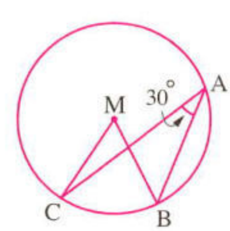
---

5 Two parallel lines to a third one are ....

(a) perpendicular. (b) parallel. (c) intersecting. (d) congruous.

---

6 **In the opposite figure :**



If m (∠ BAC) = 30°

, then m (∠ BMC) = ...

(a) 30° (b) 90°

(c) 60° (d) 120°

7 **In the opposite figure :**

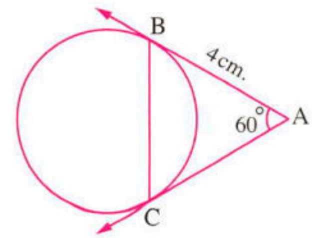
$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents

, m (∠ A) = 60° , AB = 4 cm.

, then BC = ·············· cm.

(a) 3 (b) 4

(c) 5 (d) 8



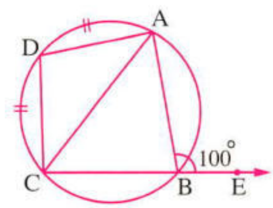
8 **In the opposite figure :**

m (∠ ABE) = 100° , m ($\overset{\frown}{AD}$) = m ($\overset{\frown}{CD}$)

, then m (∠ ACD) = ··············

(a) 100° (b) 80°

(c) 40° (d) 30°



9 The inscribed angle drawn in a semicircle is ·············· angle.

(a) an acute (b) a right (c) an obtuse (d) a straight

10 The image of the point (– 3 , 4) by reflection in the y-axis is ··············

(a) (3 , 4) (b) (3 , – 4) (c) (– 3 , – 4) (d) (4 , – 3)

11 M and N are two intersecting circles , their radii lengths are 5 cm. and 2 cm.

, then MN ∈ ··············

(a)]3 , 7[(b) [3 , 7] (c) [3 , 7[(d)]3 , 7]

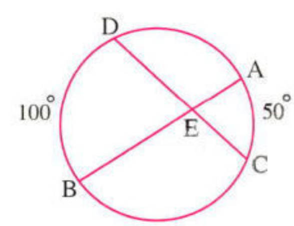
12 **In the opposite figure :**

$\overline{AB} \cap \overline{CD} = \{E\}$, m ($\overset{\frown}{AC}$) = 50° , m ($\overset{\frown}{BD}$) = 100°

, then m (∠ AEC) = ··············

(a) 50° (b) 100°

(c) 160° (d) 75°



13 The number of circles passing through a given point is ··············

(a) one circle. (b) two circles.

(c) three circles. (d) an infinite number of circles.

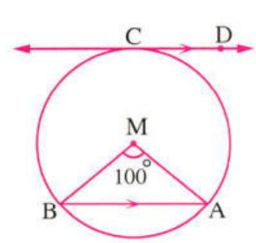
14 **In the opposite figure :**

If $\overleftrightarrow{CD}$ is a tangent of the circle M at C , $\overleftrightarrow{CD}$ // $\overline{AB}$, m (∠ M) = 100°

, then m ($\overset{\frown}{AC}$) = ··············

(a) 100° (b) 130°

(c) 260° (d) 65°

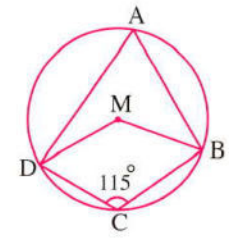


15 The numbers 5 , 4 and .... could be lengths of sides of a triangle.

(a) 8 (b) 9 (c) 10 (d) 12

---

16 **In the opposite figure :**



If m (∠ BCD) = 115°

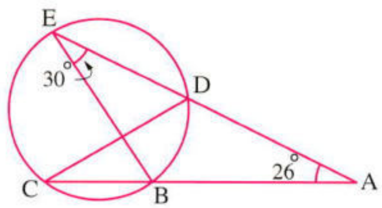
, then m (∠ BMD) = ...

(a) 65° (b) 115°

(c) 105° (d) 130°

---

17 **In the opposite figure :**



If m (∠ A) = 26° , m (∠ E) = 30°

, then m (∠ CDE) = ...

(a) 56° (b) 112°

(c) 30° (d) 82°

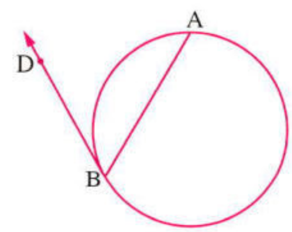
---

18 If a circle M with diameter length 14 cm. , MA = (2 X + 3) cm. where A lying on the cricle , then X = ...

(a) 5 (b) 3 (c) 2 (d) 1

---

19 **In the opposite figure :**



$\overrightarrow{BD}$ is a tangent to the circle

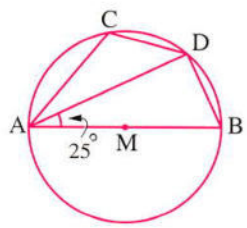
, m $(\widehat{AB}) = \frac{1}{3}$ the measure of the circle

, then m (∠ ABD) = ...

(a) 60° (b) 90° (c) 120° (d) 30°

---

20 **In the opposite figure :**



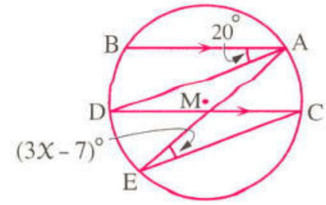
$\overline{AB}$ is a diameter in the circle M , m (∠ BAD) = 25°

, then m (∠ DCA) = ...

(a) 50° (b) 100°

(c) 115° (d) 125°

21 **In the opposite figure :**



If $\overline{AB}$ // $\overline{CD}$, m (∠ BAD) = 20°

, m (∠ AEC) = (3 X – 7)°

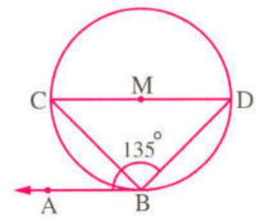
, then the value of X = ...

(a) 40° (b) 27°

(c) 20° (d) 9°

Second Essay questions

22 **In the opposite figure :**



$\overline{DC}$ is a diameter in the circle with centre M

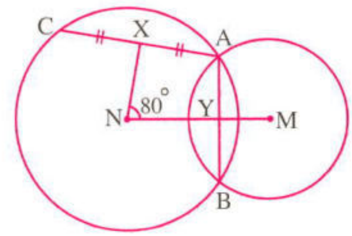
, $\overrightarrow{BA}$ is a tangent to the circle M at the point B

, m (∠ ABD) = 135°

Prove that : $\overline{DC}$ // $\overrightarrow{BA}$

---

23 **In the opposite figure :**



M and N are two intersecting circles at A and B

, $\overline{MN} \cap \overline{AB} = \{Y\}$

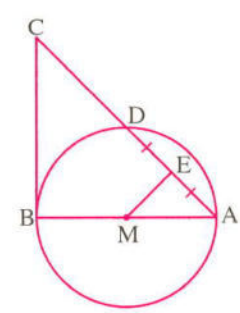
, m (∠ YNX) = 80°

, X is the midpoint of $\overline{AC}$

Find : m (∠ BAC)

---

24 **In the opposite figure :**



$\overline{AB}$ is a diameter in the circle M

, $\overline{BC}$ is a tangent-segment to it at B

, E is the midpoint of $\overline{AD}$

Prove that : the figure BMEC is a cyclic quadrilateral.

Answers of governorates' examinations of geometry

1 Cairo

1

1 c 2 a 3 c 4 b 5 b 6 d

2

[a] 1 MN = 8 + 6 = 14 cm.

2 MN = 8 – 6 = 2 cm.

[b] ∵ X is the midpoint of $\overline{AB}$ ∴ $\overline{MX} \perp \overline{AB}$

, ∵ Y is the midpoint of $\overline{CD}$ ∴ $\overline{MY} \perp \overline{CD}$

, ∵ AB = CD ∴ MX = MY

∴ In Δ MXY :

m (∠ MXY) = m (∠ MYX) = $\frac{180° - 130°}{2}$ = 25° (The req.)

3

[a] ∵ m (∠ ADB) = $\frac{1}{2}$ m (∠ AMB)

(inscribed and central angles subtended by $\widehat{AB}$)

∴ m (∠ ADB) = $\frac{1}{2}$ × 140° = 70° (First req.)

, ∵ $\overline{DB}$ // $\overline{AC}$, $\overleftrightarrow{DA}$ is a transversal

∴ m (∠ CAD) + m (∠ ADB) = 180°

(two interior angles in the same side of the transversal)

∴ m (∠ CAD) = 180° – 70° = 110° (Second req.)

[b] ∵ The measure of the arc = the measure of the central angle = 120°

∴ The length of the arc = $\frac{120°}{360°}$ × 132 = 44 cm. (The req.)

4

[a] ∵ E is the midpoint of $\overline{BC}$

∴ $\overline{ME} \perp \overline{BC}$ ∴ m (∠ MEB) = 90°

, ∵ $\overline{MD} \perp \overline{AB}$ ∴ m (∠ MDB) = 90°

∴ m (∠ MEB) + m (∠ MDB) = 90° + 90° = 180°

∴ EMDB is a cyclic quadrilateral (First req.)

, m (∠ EMD) = 360° – (90° + 90° + 56°) = 124° (Second req.)

[b] ∵ $\overrightarrow{HB}$, $\overrightarrow{HA}$ are two tangents to the circle

∴ HB = HA

∴ In Δ HAB : m (∠ HBA) = m (∠ HAB)

= $\frac{180° - 70°}{2}$ = 55°

∴ m (∠ ACB) (inscribed)

= m (∠ HAB) (tangency) = 55° (First req.)

, ∵ ABDC is a cyclic quadrilateral

∴ m (∠ CDB) + m (∠ BAC) = 180°

∴ m (∠ BAC) = 180° – 125° = 55°

∴ In Δ ABC : m (∠ ACB) = m (∠ BAC) = 55°

∴ BA = BC (Second req.)

5

[a] ∵ ∠ ABE is an exterior angle of the cyclic quadrilateral ABCD

∴ m (∠ CDA) = m (∠ ABE) = 100° (First req.)

∴ In Δ ACD : m (∠ ACD) = 180° – (100° + 50°)

= 30°

∴ m ($\widehat{DA}$) = 2 m (∠ ACD) = 60° (Second req.)

[b] ∵ $\overline{AB}$, $\overline{AD}$ are two tangent-segments to the circle M

∴ AB = AD = 4 cm.

, ∵ $\overline{AD}$, $\overline{AC}$ are two tangent-segments to the circle N

∴ AD = AC = 4 cm.

∴ The perimeter of the figure ABDC

= 4 + 3.5 + 2 + 4 = 13.5 cm. (The req.)

2 Giza

1

1 a 2 b 3 a 4 c 5 b 6 b

2

[a] In the greater circle : ∵ $\overline{ME} \perp \overline{AB}$

∴ E is the midpoint of $\overline{AB}$

∴ AE = BE (1)

, in the smaller circle : ∵ $\overline{ME} \perp \overline{CD}$

∴ E is the midpoint of $\overline{CD}$

∴ CE = DE (2)

Subtracting (2) from (1) : ∴ AC = BD (Q.E.D.)

[b] $\because \overrightarrow{XA}$, $\overrightarrow{XB}$ are two tangents to the circle

$\therefore XA = XB$

$\therefore$ In Δ XAB :

$m(\angle XAB) = m(\angle XBA) = \frac{180^\circ - 70^\circ}{2} = 55^\circ$

, $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle BAD) + m(\angle C) = 180^\circ$

$\therefore m(\angle BAD) = 180^\circ - 125^\circ = 55^\circ$

$\therefore m(\angle XAB) = m(\angle BAD)$

$\therefore \overrightarrow{AB}$ bisects $\angle DAX$ (Q.E.D. 1)

, $\because m(\angle XBA) = m(\angle BAD) = 55^\circ$

and they are alternate angles

$\therefore \overleftrightarrow{AD} \parallel \overleftrightarrow{XB}$ (Q.E.D. 2)

3

[a] $\because$ X is the midpoint of $\overline{AB}$ $\quad \therefore \overline{MX} \perp \overline{AB}$

, $\because \overrightarrow{MY} \perp \overline{AC}$, $AB = AC$

$\therefore MX = MY$ (1)

$\therefore$ In Δ MXY : $m(\angle MXY) = m(\angle MYX)$

(Q.E.D.1)

, $\because MD = ME = r$ (2)

Subtracting (1) from (2) : $\therefore XD = YE$ (Q.E.D.2)

[b] In $\Delta\Delta$ AED , AEC :

$\begin{cases} AD = AC \\ \overline{AE} \text{ is a common side} \\ m(\angle DAE) = m(\angle CAE) \end{cases}$

$\therefore \Delta AED \equiv \Delta AEC$ $\quad \therefore m(\angle ADE) = m(\angle C)$

, $\because m(\angle C) = m(\angle H)$

(two inscribed angles subtended by $\widehat{AB}$)

$\therefore m(\angle H) = m(\angle ADE)$

$\therefore$ BDEH is a cyclic quadrialteral. (Q.E.D.)

4

[a] $\because m(\widehat{AC}) = 2\, m(\angle ABC) = 2 \times 40^\circ = 80^\circ$

, $\because$ D is the midpoint of $\widehat{AC}$

$\therefore m(\widehat{AD}) = m(\widehat{DC}) = 40^\circ$

, $\because \overline{AB}$ is a diameter $\quad \therefore m(\widehat{AB}) = 180^\circ$

$\therefore m(\widehat{DCB}) = 180^\circ - 40^\circ = 140^\circ$

$\therefore m(\angle BAD) = \frac{1}{2}\, m(\widehat{DCB}) = 70^\circ$ (The req.)

[b] $\because m(\angle AMB) = 2\, m(\angle ACB) = 2 \times 45^\circ = 90^\circ$

(central and inscribed angles subtended by $\widehat{AB}$)

In Δ ABM : $\because MA = MB = r$

$\therefore m(\angle MAB) = m(\angle MBA) = \frac{180^\circ - 90^\circ}{2} = 45^\circ$

(The req.)

5

[a] $\because \overline{AB}$ is a diameter. $\quad \therefore m(\widehat{AB}) = 180^\circ$

$\therefore m(\widehat{BC}) = 180^\circ - (80^\circ + 60^\circ) = 40^\circ$

$\therefore m(\angle A) = \frac{1}{2}\, m(\widehat{BD}) = \frac{1}{2}(60^\circ + 40^\circ) = 50^\circ$

, $m(\angle B) = \frac{1}{2}\, m(\widehat{AC}) = \frac{1}{2}(60^\circ + 80^\circ) = 70^\circ$

, $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle C) = 180^\circ - m(\angle A) = 180^\circ - 50^\circ = 130^\circ$

, $m(\angle D) = 180^\circ - m(\angle B) = 180^\circ - 70^\circ = 110^\circ$

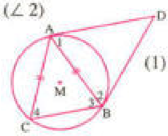
(The req.)

[b] $\because \overline{DA}$, $\overline{DB}$ are two tangent-segments

$\therefore DA = DB$

$\therefore$ In Δ ABD : $m(\angle 1) = m(\angle 2)$

$\therefore m(\angle D) = 180^\circ - 2\, m(\angle 2)$ (1)



In Δ ABC : $\because AB = AC$

$\therefore m(\angle 3) = m(\angle 4)$

$\therefore m(\angle BAC) = 180^\circ - 2\, m(\angle 4)$ (2)

, $\because m(\angle 4)$ (inscribed) $= m(\angle 2)$ (tangencey) (3)

From (1) , (2) and (3) : $\therefore m(\angle D) = m(\angle BAC)$

$\therefore \overleftrightarrow{AC}$ is a tangent to the circumcircle of Δ ABD

(Q.E.D.)

3 Alexandria

1

1 a	2 b	3 d	4 b	5 b	6 c

2

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\quad \therefore m(\angle MXA) = 90^\circ$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$ $\quad \therefore m(\angle MYA) = 90^\circ$

From the quadrilateral AXMY :

$\therefore m(\angle DME) = 360^\circ - (90^\circ + 90^\circ + 70^\circ) = 110^\circ$

(First req.)

, $\because MD = ME = r$, $XD = YE$

By subtracting : $\therefore MX = MY$

$\therefore AB = AC$ (Second req.)

[b] $\because \overleftrightarrow{AB}$ is a tangent to the circle M at D

$, \overline{CD}$ is a diameter $\quad \therefore \overline{CD} \perp \overline{AB}$

In ΔABC : $\because CA = CB \quad , \quad \overline{CD} \perp \overline{AB}$

$\therefore$ D is the midpoint of $\overline{AB}$

$\therefore AB = 2\,AD = 20$ cm. $, \because m(\angle A) = 60^\circ$

$\therefore \Delta ABC$ is an equilateral triangle.

$\therefore CA = CB = AB = 20$ cm.

$\therefore$ The perimeter of $\Delta ABC = 20 + 20 + 20$

$= 60$ cm. (The req.)

3

[a] $\because m(\angle ACB) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles subtended by $\overset{\frown}{AB}$)

$\therefore m(\angle ACB) = \frac{1}{2} \times 120^\circ = 60^\circ$ (1)

$, \because \overleftrightarrow{CD} \parallel \overline{AB} \quad \therefore m(\overset{\frown}{CA}) = m(\overset{\frown}{CB})$

$\therefore CA = CB$ (2)

From (1) and (2) :

$\therefore \Delta ABC$ is an equilateral triangle. (Q.E.D.)

[b] In ΔADE : $\because EA = ED$

$\therefore m(\angle A) = m(\angle D)$

$, \because m(\angle C) = m(\angle A)$

(inscribed angles subtended by $\overset{\frown}{BD}$)

$, m(\angle B) = m(\angle D)$

(inscribed angles subtended by $\overset{\frown}{AC}$)

$\therefore m(\angle C) = m(\angle B)$

$\therefore$ In ΔEBC : $EB = EC$ (Q.E.D.)

4

[a] In ΔXYZ : $\because XY = XZ$

$\therefore m(\angle XYZ) = m(\angle XZY) = 30^\circ$

$\therefore m(\angle X) = 180^\circ - (2 \times 30^\circ) = 120^\circ$

$\therefore m(\angle X) = m(\angle ELY) = 120^\circ$

$\therefore$ YXZL is a cyclic quadrilateral. (Q.E.D.)

[b] $\because m(\angle B) = \frac{1}{2} [m(\overset{\frown}{AC}) - m(\overset{\frown}{DE})]$

$\therefore 30^\circ = \frac{1}{2} [100^\circ - m(\overset{\frown}{DE})]$

$\therefore 60^\circ = 100^\circ - m(\overset{\frown}{DE})$

$\therefore m(\overset{\frown}{DE}) = 100^\circ - 60^\circ = 40^\circ$ (The req.)

5

[a] $\because \overrightarrow{AB} , \overrightarrow{AC}$ are two tangents to the circle.

$\therefore AB = AC$

$\therefore$ In ΔABC : $m(\angle ABC) = m(\angle ACB) = 55^\circ$

$\therefore m(\angle A) = 180^\circ - 2 \times 55^\circ = 70^\circ$ (First req.)

$, \because m(\angle CEB)$ (incribed) $= m(\angle ABC)$ (tangency)

$= 55^\circ$

$, \because$ the figure BCDE is a cyclic quadrilateral

$\therefore m(\angle CBE) + m(\angle CDE) = 180^\circ$

$\therefore m(\angle CBE) = 180^\circ - 125^\circ = 55^\circ$

$\therefore$ In ΔCBE : $m(\angle CEB) = m(\angle CBE) = 55^\circ$

$\therefore CB = CE$ (Second req.)

[b] $\because \overline{XY} \parallel \overline{BC} , \overleftrightarrow{AC}$ is a transversal.

$\therefore m(\angle AYX) = m(\angle C)$ (corresponding angles)

$, \because m(\angle C)$ (inscribed) $= m(\angle BAD)$ (tangency)

$\therefore m(\angle AYX) = m(\angle XAD)$

$\therefore \overrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

4 El-Kalyoubia

1

1 b 2 b 3 c 4 b 5 d 6 d

2

[a] $\because MA = MC = r$

$\therefore$ In ΔAMC : $m(\angle MCA) = m(\angle MAC) = 50^\circ$

$\therefore m(\angle M) = 180^\circ - 2 \times 50^\circ = 80^\circ$

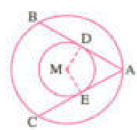
$\therefore m(\angle B) = \frac{1}{2} m(\angle M) = 40^\circ$

(inscribed and central angles subtended by $\overset{\frown}{AC}$)

$,$ in ΔABC : $\because AC = BC$

$\therefore m(\angle CAB) = m(\angle B) = 40^\circ$ (The req.)

[b] Const. : Draw $\overline{MD} , \overline{ME}$



Proof : $\because \overline{AB} , \overline{AC}$ are two tangent-segments to the smaller circle

$\therefore \overline{MD} \perp \overline{AB} , \overline{ME} \perp \overline{AC}$

$, \because MD = ME = r$ (radii of the smaller circle)

$\therefore AB = AC$ (Q.E.D.)

3

[a] $\because \overline{AE}$ is a diameter $\therefore m(\angle D) = 90^\circ$

, $\because \overline{CB} \perp \overline{AE}$ $\therefore m(\angle CBA) = 90^\circ$

$\therefore m(\angle D) + m(\angle CBA) = 90^\circ + 90^\circ = 180^\circ$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

[b] In $\Delta\Delta$ AEB , ADC : $\because m(\angle B) = m(\angle C)$

(two inscribed angles subtended by $\overset{\frown}{AD}$)

, $m(\angle BAE) = m(\angle DAC)$

$\therefore m(\angle AEB) = m(\angle ADC) = 90^\circ$

$\therefore \overline{AC}$ is a diameter in the circle. (Q.E.D.)

4

[a] $\because m(\angle BAC) = \frac{1}{2} m(\overset{\frown}{BC}) = \frac{1}{2} \times 120^\circ = 60^\circ$

$\therefore$ In ΔABC : $m(\angle C) = 180^\circ - (70^\circ + 60^\circ) = 50^\circ$

$\therefore m(\angle DAB)$ (tangency) $= m(\angle C)$ (inscribed)

$= 50^\circ$ (The req.)

[b] $m(\angle AEC) = \frac{1}{2} [m(\overset{\frown}{BD}) + m(\overset{\frown}{AC})]$

$= \frac{1}{2} (100^\circ + 50^\circ) = 75^\circ$ (The req.)

5

[a] In ΔABC : $\because m(\angle BAC) = 90^\circ$

, $AC = \frac{1}{2} BC$ $\therefore m(\angle ABC) = 30^\circ$

$\therefore m(\angle C) = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$

$\therefore m(\angle DAB) = m(\angle C) = 60^\circ$

$\therefore \overrightarrow{AD}$ is a tangent to the circle which passes through ΔABC (Q.E.D.)

[b] $\because \overline{AB}$ is a diameter $\therefore m(\overset{\frown}{AB}) = 180^\circ$

, $\because \overline{AB} \,/\!/\, \overline{CD}$

$\therefore m(\overset{\frown}{BD}) = m(\overset{\frown}{AC}) = \frac{180^\circ - 80^\circ}{2} = 50^\circ$

$\therefore m(\angle BED) = \frac{1}{2} m(\overset{\frown}{BD}) = 25^\circ$

$\therefore 3X - 20^\circ = 25^\circ$ $\therefore 3X = 45^\circ$

$\therefore X = 15^\circ$ (The req.)

5 El-Sharkia

1

1 d 2 c 3 a 4 c 5 a 6 b

2

[a] $\because$ D is the midpoint of $\overline{AB}$ $\therefore \overline{MD} \perp \overline{AB}$

, $\because \overline{MH} \perp \overline{AC}$, MD = MH $\therefore$ AB = AC

$\therefore$ In ΔABC : $m(\angle B) = m(\angle C) = 65^\circ$

$\therefore m(\angle A) = 180^\circ - 2 \times 65^\circ = 50^\circ$ (The req.)

[b] $\because m(\angle ACB) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles subtended by $\overset{\frown}{AB}$)

$\therefore m(\angle ACB) = \frac{1}{2} \times 120^\circ = 60^\circ$ (1)

, $\because \overleftrightarrow{CD} \,/\!/\, \overline{AB}$ $\therefore m(\overset{\frown}{CA}) = m(\overset{\frown}{CB})$

$\therefore$ CA = CB (2)

From (1) and (2) :

$\therefore \Delta CAB$ is an equilateral triangle. (Q.E.D.)

3

[a] $\because$ C is the midpoint of $\overline{OL}$

$\therefore \overrightarrow{MC} \perp \overline{OL}$ $\therefore m(\angle OCB) = 90^\circ$

, $\because \overline{AB}$ is a diameter. $\therefore m(\angle ADB) = 90^\circ$

$\therefore m(\angle HCB) + m(\angle HDB) = 90^\circ + 90^\circ = 180^\circ$

$\therefore$ HDBC is a cyclic quadrilateral. (Q.E.D.)

[b] $\because$ AC = BD $\therefore m(\overset{\frown}{ADC}) = m(\overset{\frown}{BAD})$

Subtracting $m(\overset{\frown}{AD})$ from both sides.

$\therefore m(\overset{\frown}{CD}) = m(\overset{\frown}{AB})$ $\therefore$ CD = AB

$\therefore X + 3 = 3X - 5$ $\therefore 2X = 8$

$\therefore X = 4$

$\therefore AB = 3 \times 4 - 5 = 7$ cm. (The req.)

4

[a] $\because \overline{AB}$ is a tangent-segment to the cirlce

$\therefore m(\angle ABC)$ tangency $= m(\angle BDC)$ inscribed

$= 70^\circ$

, $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle

$\therefore$ AB = AC

In ΔABC : $\therefore m(\angle ABC) = m(\angle ACB) = 70^\circ$

$\therefore m(\angle A) = 180^\circ - 2 \times 70^\circ = 40^\circ$ (1) (First req.)

, $\because$ HBCD is a cyclic quadrilateral.

$\therefore m(\angle BCD) + m(\angle BHD) = 180^\circ$

$\therefore m(\angle BCD) = 180° - 140° = 40°$ (2)

From (1) and (2) : $\therefore m(\angle BCD) = m(\angle A)$

$\therefore \overleftrightarrow{CD}$ is a tangent to the circle which passes through the points A , B and C (Second req.)

[b] $\because$ X is the midpoint of $\overline{AB}$ $\therefore \overline{MX} \perp \overline{AB}$

, $\because \overline{AB} // \overline{CD}$ $\therefore \overline{MY} \perp \overline{CD}$

$\therefore$ Y is the midpoint of $\overline{CD}$ (Q.E.D.)

5

[a] $\because m(\angle A) = \frac{1}{2}[m(\widehat{BC}) - m(\widehat{HD})]$

$\therefore 30° = \frac{1}{2}[100° - m(\widehat{HD})]$

$\therefore 60° = 100° - m(\widehat{HD})$

$\therefore m(\widehat{HD}) = 100° - 60° = 40°$ (The req.)

[b] In $\Delta\Delta$ ADC , HDO :

$$\begin{cases} AD = HD \\ CD = OD \\ m(\angle ADC) = m(\angle HDO) \text{ (V.O.A)} \end{cases}$$

$\therefore \Delta ADC \equiv \Delta HDO$

$\therefore m(\angle CAO) = m(\angle CHO)$

and they are drawn on $\overline{CO}$ and on one side of it

$\therefore$ ACOH is a cyclic quadrilateral. (Q.E.D. 1)

, $\because \overline{AD}$ is a tangent-segment to the circle.

$\therefore m(\angle ABC)$ (inscribed) $= m(\angle CAD)$ (tangency)

$\therefore m(\angle ABC) = m(\angle CHO)$

and they are alternate angles.

$\therefore \overline{AB} // \overline{OH}$ (Q.E.D. 2)

6 El-Monofia

1

1 c 2 a 3 d 4 b 5 c 6 b

2

[a] In the greater circle : $\because \overline{ME} \perp \overline{AB}$

$\therefore$ E is the midpoint of $\overline{AB}$ $\therefore AE = EB$ (1)

In the smaller circle : $\because \overline{ME} \perp \overline{CD}$

$\therefore$ E is the midpoint of $\overline{CD}$ $\therefore CE = DE$ (2)

Subtracting (2) from (1) : $\therefore AC = BD$ (Q.E.D.)

[b] $\because \overline{EA}$ and $\overline{EC}$ are two tangent-segments to the circle M $\therefore EA = EC$ (1)

, $\because \overline{EB}$ and $\overline{ED}$ are two tangent-segments to the circle N $\therefore EB = ED$ (2)

Subtracting (2) from (1) : $\therefore AB = CD$ (Q.E.D.)

3

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\therefore m(\angle MXA) = 90°$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$ $\therefore m(\angle MYA) = 90°$

From the quadrilateral AXMY :

$\therefore m(\angle DME) = 360° - (90° + 90° + 70°) = 110°$ (First req.)

, $\because AB = AC$, $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

$\therefore MX = MY$, $\because MD = ME = r$

By subtracting : $\therefore XD = YE$ (Second req.)

[b] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle D) = m(\angle ABE) = 100°$

In Δ ACD :

$\therefore m(\angle DCA) = 180° - (100° + 40°) = 40°$

$\therefore m(\angle DCA) = m(\angle DAC)$

$\therefore m(\widehat{AD}) = m(\widehat{CD})$ (Q.E.D.)

4

[a] $\because \overline{DE} // \overline{BC}$ $\therefore m(\widehat{DB}) = m(\widehat{EC})$

$\therefore m(\angle DAB) = m(\angle EAC)$

Adding $m(\angle BAC)$ to both sides.

$\therefore m(\angle DAC) = m(\angle BAE)$ (Q.E.D.)

[b] $\because \overline{XY} // \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle AYX) = m(\angle C)$ (corresponding angles)

, $\because m(\angle C)$ (inscribed) $= m(\angle BAD)$ (tangency)

$\therefore m(\angle AYX) = m(\angle XAD)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

5

[a] $\because \overline{CD} \cap \overline{EF} = \{A\}$

$\therefore m(\angle EAC) = m(\angle FAD)$ (V.O.A.) (1)

$, \because m(\angle EBC) = m(\angle EAC)$ (2)

(two inscribed angles subtended by $\widehat{EC}$)

$, \because m(\angle FBD) = m(\angle FAD)$ (3)

(two inscribed angles subtended by $\widehat{FD}$)

From (1) , (2) , (3) :

$\therefore m(\angle EBC) = m(\angle FBD)$ (Q.E.D.)

[b] $\because m(\angle BAC) = m(\angle BDC)$

(two inscribed angles subtended by $\widehat{BC}$)

$\therefore \frac{1}{2} m(\angle BAC) = \frac{1}{2} m(\angle BDC)$

$\therefore m(\angle XAY) = m(\angle XDY)$ and they are drawn on $\overline{XY}$ and on one side of it.

$\therefore$ AXYD is a cyclic quadrilateral. (Q.E.D.)

7 El-Gharbia

1

1 d 2 a 3 d 4 b 5 a 6 c

2

[a] $\because \overline{MD} \perp \overline{BC}$ $\therefore$ D is the midpoint of $\overline{BC}$

$\therefore DC = \frac{1}{2} BC$ (1)

$, \because \overline{ME} \perp \overline{AC}$ $\therefore$ E is the midpoint of $\overline{AC}$

$\therefore EC = \frac{1}{2} AC$ (2)

$, \because$ D and E are the two midpoints of $\overline{BC}$ and $\overline{AC}$ respectively

$\therefore DE = \frac{1}{2} AB$ (3)

Adding (1) , (2) and (3) :

$\therefore$ The perimeter of Δ CDE $= \frac{1}{2}$ the perimeter of Δ ABC (Q.E.D.)

[b] $\because m(\angle ADC) = m(\angle AEC) = 25°$

(two inscribed angles subtended by $\widehat{AC}$)

$, \because \overline{AB} \parallel \overline{CD}$, $\overleftrightarrow{AD}$ is a transversal

$\therefore m(\angle ADC) = m(\angle BAD)$

$= 25°$ (alternate angles)

$\therefore 3X - 5 = 25°$ $\therefore 3X = 30°$

$\therefore X = 10°$ (The req.)

3

[a] $\because \overline{XY} \parallel \overrightarrow{BD}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore m(\angle DBX) = m(\angle BXY)$ (alternate angles)

$, \because m(\angle C)$ (inscribed) $= m(\angle ABD)$ (tangency)

$\therefore m(\angle C) = m(\angle BXY)$

$\therefore$ AXYC is a cyclic quadrilateral. (Q.E.D.)

[b] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle A) + m(\angle C) = 180°$

$\therefore m(\angle A) = 180° - 70° = 110°$

$\therefore$ In Δ ABD : $m(\angle ABD) = 180° - (110° + 30°)$

$= 40°$ (The req.)

4

[a] $\because m(\angle CMD) = 70°$ $\therefore m(\widehat{CD}) = 70°$

$, \because \overline{AB}$ is a diameter.

$\therefore m(\widehat{AC}) + m(\widehat{CD}) + m(\widehat{DB}) = 180°$

$\therefore m(\widehat{AC}) + m(\widehat{DB}) = 180° - 70° = 110°$

Let $m(\widehat{AC}) = 5X$, $m(\widehat{BD}) = 6X$

$\therefore 5X + 6X = 110°$ $\therefore 11X = 110°$ $\therefore X = 10°$

$\therefore m(\widehat{AC}) = 5 \times 10° = 50°$

$\therefore m(\widehat{ACD}) = 50° + 70° = 120°$ (The req.)

[b] $\because \overrightarrow{XA}$, $\overrightarrow{XB}$ are two tangents to the circle

$\therefore XA = XB$

$\therefore$ In Δ ABX : $m(\angle XAB) = m(\angle XBA)$

$= \frac{180° - 80°}{2} = 50°$ (1)

$, \because$ The figure ABCD is a cyclic quadrilateral

$\therefore m(\angle BAD) + m(\angle BCD) = 180°$

$\therefore m(\angle BAD) = 180° - 130° = 50°$ (2)

From (1) and (2) : $\therefore m(\angle BAD) = m(\angle XBA)$ and they are alternate angles.

$\therefore \overline{AD} \parallel \overrightarrow{XB}$ (Q.E.D.)

5

[a] In Δ ABD : $\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB) = \frac{180° - 70°}{2} = 55°$

$\therefore m(\angle DBC) = 125° - 55° = 70°$

$\therefore m(\angle DBC) = m(\angle A)$

$\therefore \overline{BC}$ is a tangent-segment to the circle passing through the points A , B and D (Q.E.D.)

[b] $\because \overleftrightarrow{MN}$ is the line of centres

, $\overline{AB}$ is the common chord

$\therefore \overleftrightarrow{MN} \perp \overline{AB}$ $\quad \therefore m(\angle AEN) = 90°$

, $\because \overline{CD}$ is a tangent-segment to the circle N at D

$\therefore \overline{ND} \perp \overline{CD}$ $\quad \therefore m(\angle CDN) = 90°$

From the quadrilateral CDNE :

$\therefore m(\angle DCE) = 360° - (125° + 90° + 90°) = 55°$

(The req.)

8 El-Dakahlia

1

[a] 1 b $\quad$ 2 c $\quad$ 3 d

[b] $\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$ $\quad \therefore m(\angle MDA) = 90°$

, $\because \overline{ME} \perp \overline{AC}$ $\quad \therefore m(\angle MEA) = 90°$

From the quadrilateral ADME :

$\therefore m(\angle A) = 360° - (90° + 90° + 120°) = 60°$

, $\because MD = ME$ $\quad \therefore AB = AC$

, $\because m(\angle A) = 60°$

$\therefore \Delta ABC$ is an equilateral triangle. (Q.E.D.)

2

[a] 1 a $\quad$ 2 d $\quad$ 3 a

[b] $\because$ D is the midpoint of $\overline{AC}$

$\therefore \overline{MD} \perp \overline{AC}$ $\quad \therefore m(\angle MDA) = 90°$

$\therefore$ In ΔMDA : $(AD)^2 = (MA)^2 - (MD)^2 = 5^2 - 3^2 = 16$

$\therefore AD = \sqrt{16} = 4$ cm.

$\therefore$ The area of $\Delta MDA = \frac{1}{2} \times AD \times MD = \frac{1}{2} \times 4 \times 3 = 6 \text{ cm}^2$

, $\because \overline{AB}$ is a tangent-segment to the circle

$\therefore \overline{MA} \perp \overline{AB}$ $\quad \therefore m(\angle MAB) = 90°$

$\therefore$ The area of $\Delta MAB = \frac{1}{2} \times AB \times MA = \frac{1}{2} \times 12 \times 5 = 30 \text{ cm}^2$

$\therefore$ The area of the figure ABMD

= The area of ΔMDA + the area of ΔMAB

$= 6 + 30 = 36 \text{ cm}^2$ (The req.)

3

[a] $\because m(\angle A) = \frac{1}{2}[m(\overset{\frown}{EC}) - m(\overset{\frown}{BD})]$

$\therefore 40° = \frac{1}{2}[m(\overset{\frown}{EC}) - 60°]$

$\therefore 80° = m(\overset{\frown}{EC}) - 60°$

$\therefore m(\overset{\frown}{EC}) = 80° + 60° = 140°$

, $\because CB = ED$

$\therefore m(\overset{\frown}{BC}) = m(\overset{\frown}{ED}) = \frac{360° - (140° + 60°)}{2} = 80°$

(The req.)

[b] $\because$ ABFE is a cyclic quadrilateral

$\therefore m(\angle E) + m(\angle ABF) = 180°$ (1)

, $\because \angle ABF$ is an exterior angle of the cyclic quadrilateral ABCD

$\therefore m(\angle ABF) = m(\angle D)$ (2)

Substituting from (2) in (1) :

$\therefore m(\angle E) + m(\angle D) = 180°$

$\therefore 2X° + 7X° = 180°$

$\therefore 9X° = 180°$ $\quad \therefore X = 20°$ (The req.)

4

[a] In the circle N :

$m(\angle EAF) = \frac{1}{2} m(\angle ENF) = \frac{1}{2} \times 50° = 25°$

(inscribed and central angles subtended by $\overset{\frown}{EF}$)

, $\because m(\angle CAD) = m(\angle EAF) = 25°$ (V.O.A.)

$\therefore$ In the circle M : $m(\angle DBC) = m(\angle CAD) = 25°$

(two inscribed angles subtended by $\overset{\frown}{CD}$)

(The req.)

[b] $\because \overline{BY}$, $\overline{BX}$ are two tangent-segments

$\therefore BY = BX$

, $\because \overline{CY}$, $\overline{CZ}$ are two tangent-segments

$\therefore CY = CZ$

, $\because BY + CY = BC = 10$ cm. $\quad \therefore BX + CZ = 10$ cm.

, $\because \overline{AX}$, $\overline{AZ}$ are two tangent-segments.

$\therefore AX = AZ$

, $\because$ the perimeter of $\Delta ABC = 30$ cm.

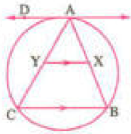
$\therefore AX + AZ + BY + BX + CY + CZ = 30$ cm.

$\therefore 2AX + 10 + 10 = 30$ $\quad \therefore 2AX + 20 = 30$

$\therefore 2X = 10$

$\therefore AX = 5$ cm. (The req.)

5



[a] $\because \overleftrightarrow{AD}$ is a tangent to the circle

$\therefore$ m ($\angle$ DAC) (tangency)

= m ($\angle$ B) (inscribed)

, $\because \overline{XY} \parallel \overline{BC}$

, $\overleftrightarrow{AB}$ is a transversal

$\therefore$ m ($\angle$ AXY) = m ($\angle$ B) (corresponding angles)

$\therefore$ m ($\angle$ AXY) = m ($\angle$ DAC)

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

[b] $\because$ m ($\angle$ ADB) = m ($\angle$ AEB) = 90°

and they are drawn on $\overline{AB}$ and on one side of it

$\therefore$ ABDE is a cyclic quadrilateral

$\therefore$ m ($\angle$ BAD) = m ($\angle$ BED) (1)

In Δ NAB : $\because$ X is the midpoint of $\overline{NB}$

, Y is the midpoint of $\overline{NA}$ $\therefore \overline{XY} \parallel \overline{AB}$

$\therefore$ m ($\angle$ BAD) = m ($\angle$ XYD)

(corresponding angles) (2)

From (1) and (2) : $\therefore$ m ($\angle$ XED) = m ($\angle$ XYD)

and they are drawn on $\overline{XD}$ and on one side of it.

$\therefore$ XYED is a cyclic quadrilateral. (Q.E.D.)

9 Ismailia

1

1 c 2 b 3 c 4 d 5 a 6 b

2

[a] $\because$ m ($\angle$ BAC) = m ($\angle$ BDC) = 70°

(two inscribed angles subtended by $\overset{\frown}{BC}$)

, $\because \overline{BD} \cap \overline{AC}$ = {E}

$\therefore \angle$ AED is an exterior angle of Δ AEB

$\therefore$ m ($\angle$ B) = m ($\angle$ AED) – m ($\angle$ A)

= 110° – 70° = 40° (The req.)

[b] $\because$ ABCD is a cyclic quadrilateral.

$\therefore$ m ($\angle$ B) + m ($\angle$ D) = 180°

$\therefore$ m ($\angle$ B) = 180° – 120° = 60°

, $\because \overline{AB}$ is a diameter $\therefore$ m ($\angle$ ACB) = 90°

$\therefore$ In Δ ACB : m ($\angle$ CAB) = 180° – (90° + 60°)

= 30° (The req.)

3

[a] $\because$ m ($\angle$ A) = $\frac{1}{2}$ m ($\angle$ M) = X

(inscribed and central angles subtended by $\overset{\frown}{BD}$)

, $\because$ ABCD is a cyclic quadrilateral.

$\therefore$ m ($\angle$ A) + m ($\angle$ C) = 180°

$\therefore$ X° + 2 X° = 180°

$\therefore$ 3 X° = 180° $\therefore$ X° = 60°

$\therefore$ m ($\angle$ A) = 60° (The req.)

[b] In Δ ABC : $\because$ m ($\angle$ C) = 180° – (70° + 40°) = 70°

$\therefore$ m ($\angle$ B) = m ($\angle$ C) $\therefore$ AB = AC

, $\because \overline{MD} \perp \overline{AB}$, $\overline{ME} \perp \overline{AC}$

$\therefore$ MD = ME (Q.E.D.)

4

[a] $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents. $\therefore$ AB = AC

$\therefore$ In Δ ABC : m ($\angle$ ABC) = m ($\angle$ ACB)

$= \frac{180° - 50°}{2} = 65°$

$\therefore$ m ($\angle$ CDB) (inscribed)

= m ($\angle$ ABC) (tangency) = 65° (First req.)

, $\because \overrightarrow{AB} \parallel \overline{CD}$, $\overleftrightarrow{BC}$ is a transversal

$\therefore$ m ($\angle$ ABC) = m ($\angle$ BCD) = 65° (alternate angles)

$\therefore$ In Δ BCD : m ($\angle$ CBD) = 180° – (65° + 65°)

= 50°

$\therefore$ m ($\angle$ CBD) = m ($\angle$ A)

$\therefore \overline{BD}$ is a tangent-segment to the circle passing through the vertices of Δ ABC (Second req.)

[b] $\because \overline{AB}$ is a tangent-segment to the circle M at A

$\therefore \overline{MA} \perp \overline{AB}$ $\therefore$ m ($\angle$ MAB) = 90°

$\therefore (MB)^2 = (MA)^2 + (AB)^2 = 6^2 + 8^2 = 100$

$\therefore$ MB = $\sqrt{100}$ = 10 cm.

, $\because$ MD = r = 6 cm.

$\therefore$ BD = 10 – 6 = 4 cm. (The req.)

5

[a] $\because \overrightarrow{XY} \parallel \overline{DE}$, $\overleftrightarrow{XZ}$ is a transversal

$\therefore$ m ($\angle$ XED) = m ($\angle$ EXY) (alternate angles)

, $\because$ m ($\angle$ L) (inscribed) = m ($\angle$ ZXY) (tangency)

$\therefore$ m ($\angle$ L) = m ($\angle$ XED)

$\therefore$ EDLZ is a cyclic quadrilateral (Q.E.D.)

[b] $\because$ AB = CD $\quad\therefore$ m $(\widehat{ADB})$ = m $(\widehat{CBD})$

Subtracting m $(\widehat{BD})$ from both sides

$\therefore$ m $(\widehat{AD})$ = m $(\widehat{CB})$ $\quad\therefore$ AD = CB (Q.E.D.)

10 Damietta

1

1 c 2 c 3 a 4 d 5 b 6 c

2

[a] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\quad\therefore$ m ($\angle$ MXA) = 90°

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$ $\quad\therefore$ m ($\angle$ MYA) = 90°

From the quadrilateral AXMY :

$\therefore$ m ($\angle$ DME) = 360° – (90° + 90° + 70°) = 110° (First req.)

, $\because$ AB = AC , $\overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

$\therefore$ MX = MY $\quad$, $\because$ MD = ME = r

By subtracting : $\therefore$ XD = YE (Second req.)

[b] $\because \overline{XY} \,//\, \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore$ m ($\angle$ AYX) = m ($\angle$ ACB) (corresponding angles)

, $\because$ m ($\angle$ ACB) (inscribed) = m ($\angle$ BAD) (tangency)

$\therefore$ m ($\angle$ AYX) = m ($\angle$ XAD)

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

3

[a] In Δ ABM : $\because$ MA = MB = r

$\therefore$ m ($\angle$ MAB) = m ($\angle$ MBA) = 50°

$\therefore$ m ($\angle$ M) = 180° – 2 × 50° = 80°

$\therefore$ m ($\angle$ C) = $\frac{1}{2}$ m ($\angle$ M) = 40°

(inscribed and central angles subtended by $\widehat{AB}$) (First req.)

, $\because$ m $(\widehat{AB})$ = m ($\angle$ M) = 80°

$\therefore$ m $(\widehat{AC})$ + m $(\widehat{BC})$ = 360° – 80° = 280°

, $\because$ m $(\widehat{AC})$ = m $(\widehat{BC})$

$\therefore$ m $(\widehat{AC}) = \frac{280°}{2}$ = 140° (Second req.)

[b] $\because$ ABCD is a cyclic quadrilateral.

$\therefore$ m ($\angle$ A) + m ($\angle$ C) = 180°

$\therefore$ m ($\angle$ C) = 180° – 120° = 60° (First req.)

, $\because \overrightarrow{BF}$ bisects $\angle$ EBC

$\therefore$ m ($\angle$ EBC) = 130° $\quad$, $\because$ E $\in \overrightarrow{AB}$

$\therefore$ m ($\angle$ D) = m ($\angle$ EBC) = 130°

(exterior angle of the cyclic quadrilateral) (Second req.)

4

[a] m ($\angle$ A) = $\frac{1}{2}$ [m $(\widehat{EC})$ – m $(\widehat{DB})$]

$= \frac{1}{2}$ (100° – 40°) = 30° (The req.)

[b] $\because \overline{BC}$ is a tangent-segment to the circle

$\therefore$ m ($\angle$ CBD) (tangency) = m ($\angle$ BAE) (inscribed)

, $\because$ E is the midpoint of $\widehat{BF}$

$\therefore$ m $(\widehat{BE})$ = m $(\widehat{FE})$

$\therefore$ m ($\angle$ BAE) = m ($\angle$ FAE)

$\therefore$ m ($\angle$ CBD) = m ($\angle$ CAD)

and they are drawn on $\overline{CD}$ and on one side of it

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

5

[a] $\because \overline{AB}$ is a tangent-segment to the circle at A

$\therefore \overline{MA} \perp \overline{AB}$ $\quad\therefore$ m ($\angle$ MAB) = 90°

$\therefore (MB)^2 = (MA)^2 + (AB)^2 = 6^2 + 8^2 = 100$

$\therefore$ MB = $\sqrt{100}$ = 10 cm. , $\because$ MD = r = 6 cm.

$\therefore$ DB = 10 – 6 = 4 cm. (The req.)

[b] $\because$ m ($\angle$ DCB) = $\frac{1}{2}$ m ($\angle$ DMB) = $\frac{1}{2}$ × 140° = 70°

(inscribed and central angles subtended by $\widehat{BD}$)

, $\because \overline{AB} \,//\, \overline{CD}$, $\overleftrightarrow{CB}$ is a transversal

$\therefore$ m ($\angle$ DCB) = m ($\angle$ ABC)

= 70° (alternate angles) (1)

, $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments

$\therefore$ AB = AC

$\therefore$ In Δ ABC : m ($\angle$ ACB) = m ($\angle$ ABC) = 70° (2)

From (1) and (2) : $\therefore$ m ($\angle$ DCB) = m ($\angle$ ACB)

$\therefore \overrightarrow{CB}$ bisects $\angle$ ACD (First req.)

, m ($\angle$ A) = 180° – 2 × 70° = 40° (Second req.)

11 El-Beheira

1

1 c 2 b 3 b 4 a 5 d 6 a

2

[a] $\because \overline{DE} \parallel \overline{BC}$ $\therefore m(\widehat{BD}) = m(\widehat{CE})$

$\therefore m(\angle DAB) = m(\angle EAC)$

Adding $m(\angle BAC)$ to both sides

$\therefore m(\angle DAC) = m(\angle EAB)$ (Q.E.D.)

[b] $\because \overleftrightarrow{MN}$ is the line of centres

, $\overline{AB}$ is the common chord

$\therefore \overleftrightarrow{MN} \perp \overline{AB}$ $\therefore m(\angle AEN) = 90^\circ$

From the quadrilateral CEND :

$m(\angle CDN) = 360^\circ - (90^\circ + 125^\circ + 55^\circ) = 90^\circ$

$\therefore \overleftrightarrow{CD}$ is a tangent to the circle N at D (Q.E.D.)

3

[a] $\because m(\angle DCB) = \frac{1}{2} m(\angle DMB) = 65^\circ$ (1)

(inscribed and central angles subtended by $\widehat{BD}$)

, $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments

$\therefore AB = AC$

$\therefore$ In ΔABC : $m(\angle ABC) = m(\angle ACB$

$= \frac{180^\circ - 50^\circ}{2} = 65^\circ$ (2)

From (1) and (2) : $\therefore m(\angle DCB) = m(\angle ABC)$

and they are alternate angles

$\therefore \overleftrightarrow{AB} \parallel \overline{CD}$ (Q.E.D.)

[b] In ΔABC : $\because m(\angle B) = m(\angle C)$

$\therefore AB = AC$

, $\because \overrightarrow{MX} \perp \overline{AB}$, $\overrightarrow{MY} \perp \overline{AC}$

$\therefore MX = MY$, $\because MD = ME = r$

By subtracting : $\therefore XD = YE$ (Q.E.D.)

4

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle A) + m(\angle C) = 180^\circ$

$\therefore m(\angle A) = 180^\circ - 140^\circ = 40^\circ$ (First req.)

, $\because \overline{AB}$ is a diameter $\therefore m(\angle ADB) = 90^\circ$

In ΔBCD : $\because CB = CD$

$\therefore m(\angle CBD) = m(\angle CDB) = \frac{180^\circ - 140^\circ}{2} = 20^\circ$

$\therefore m(\angle ADC) = 90^\circ + 20^\circ = 110^\circ$ (Second req.)

[b] $\because \overline{XY} \parallel \overline{BC}$, $\overleftrightarrow{AC}$ is a transversal

$\therefore m(\angle AYX) = m(\angle ACB)$ (corresponding angles)

, $\because m(\angle ACB)$ (inscribed) $= m(\angle BAD)$ (tangency)

$\therefore m(\angle AYX) = m(\angle XAD)$

$\therefore \overleftrightarrow{AD}$ is a tangent to the circle passing through the points A , X and Y (Q.E.D.)

5

[a] $\because m(\widehat{AB}) = m(\angle M) = 108^\circ$

$\therefore$ The length of $\widehat{AB} = \frac{m(\widehat{AB})}{360^\circ} \times 2\pi r$

$= \frac{108^\circ}{360^\circ} \times 2 \times 3.14 \times 5$

$= 9.42$ cm. (The req.)

[b] $\because$ X is the midpoint of $\overline{AC}$

$\therefore \overline{MX} \perp \overline{AC}$ $\therefore m(\angle AXY) = 90^\circ$

, $\because \overline{BY}$ is a tangent-segment at B

$\therefore \overline{MB} \perp \overline{BY}$ $\therefore m(\angle ABY) = 90^\circ$

$\therefore m(\angle AXY) = m(\angle ABY) = 90^\circ$

and they are drawn on $\overline{AY}$ and on one side of it

$\therefore$ AXBY is a cyclic quadrilatral. (Q.E.D.)

12 El-Menia

1

1 d 2 d 3 c 4 b 5 b 6 b

2

[a] $\because r_1 = 9$ cm. , $r_2 = 4$ cm. , $r_1 - r_2 = 5$ cm.

, $r_1 + r_2 = 13$ cm.

1 $\because MN = 10$ cm. $\therefore r_1 - r_2 < MN < r_1 + r_2$

$\therefore$ The two circles are intersecting.

2 $\because MN = 5$ cm. $\therefore MN = r_1 - r_2$

$\therefore$ The two circles are touching internally.

3 $\because MN = 3$ cm. $\therefore MN < r_1 - r_2$

$\therefore$ One of the two circles is inside the other.

(The circle N is inside the circle M)

[b] $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$ $\therefore m(\angle AXM) = 90^\circ$

, $\because$ Y is the midpoint of $\overline{AC}$

$\therefore \overline{MY} \perp \overline{AC}$ $\therefore m(\angle AYM) = 90^\circ$

From the quadrilateral AYMX :

$\therefore m(\angle XMY) = 360^\circ - (90^\circ + 90^\circ + 50^\circ)$

$= 130^\circ$ (First req.)

In Δ ABC : $\because m(\angle C) = 180^\circ - (50^\circ + 65^\circ) = 65^\circ$

$\therefore m(\angle B) = m(\angle C) = 65^\circ$

$\therefore AB = AC$ $\therefore MX = MY$ (Second req.)

3

[a] $\because m(\widehat{AC}) = 2\, m(\angle B) = 2 \times 36^\circ = 72^\circ$

$\therefore$ The length of $\widehat{AC} = \frac{m(\widehat{AC})}{360^\circ} \times 2\pi r$

$= \frac{72^\circ}{360^\circ} \times 2 \times 3.14 \times 5$

$= 6.28$ cm. (The req.)

[b] $\because m(\angle ABD) = \frac{1}{2} m(\angle AMD)$

(inscribed and central angles subtended by $\widehat{AD}$)

$\therefore m(\angle ABD) = \frac{1}{2} \times 60^\circ = 30^\circ$ (First req.)

, $\because \overline{DC} \parallel \overline{AB}$, $\overleftrightarrow{BD}$ is a transversal.

$\therefore m(\angle BDC) = m(\angle ABD) = 30^\circ$

(alternate angles)

, $\because \overline{CD}$ is a diameter $\therefore m(\angle CBD) = 90^\circ$

$\therefore$ In Δ CBD : $m(\angle BCD) = 180^\circ - (90^\circ + 30^\circ)$

$= 60^\circ$ (Second req.)

4

[a] State by yourself.

[b] In Δ ABD : $\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB) = 30^\circ$

$\therefore m(\angle A) = 180^\circ - 2 \times 30^\circ = 120^\circ$

$\therefore m(\angle A) + m(\angle C) = 120^\circ + 60^\circ = 180^\circ$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

5

[a] In Δ ABC : $\because m(\angle BAC) = 90^\circ$

$\therefore m(\angle B) + m(\angle C) = 90^\circ$ (1)

In Δ ADB : $\because m(\angle ADB) = 90^\circ$

$\therefore m(\angle B) + m(\angle DAB) = 90^\circ$ (2)

From (1) and (2) : $\therefore m(\angle C) = m(\angle DAB)$

$\therefore \overleftrightarrow{AB}$ is a tangent to the circle passing through the points A , C and D (Q.E.D.)

[b] $\because \overrightarrow{AB}$ is a tangent to the circle

$\therefore m(\angle ABC)$ (tangency) $= m(\angle BDC)$ (inscribed)

$= 70^\circ$

, $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle.

$\therefore AB = AC$

$\therefore$ In Δ ABC : $m(\angle ACB) = m(\angle ABC) = 70^\circ$

$\therefore m(\angle CAB) = 180^\circ - 2 \times 70^\circ = 40^\circ$ (The req.)

13 Assiut

1

1 d	2 c	3 a	4 b	5 d	6 b

2

[a] In Δ ABC : $\because m(\angle B) = m(\angle C)$

$\therefore AB = AC$

, $\because$ X is the midpoint of $\overline{AB}$

$\therefore \overline{MX} \perp \overline{AB}$, $\because \overline{MY} \perp \overline{AC}$

$\therefore MX = MY$ (Q.E.D.)

[b] $\because m(\angle ADB) = \frac{1}{2} m(\widehat{AB}) = \frac{1}{2} \times 110^\circ = 55^\circ$

, $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle ADC) = m(\angle CBE) = 85^\circ$

$\therefore m(\angle BDC) = 85^\circ - 55^\circ = 30^\circ$ (The req.)

3

[a] $\because m(\angle CMD) = 70^\circ$ $\therefore m(\widehat{CD}) = 70^\circ$

, $\because \overline{AB}$ is a diameter

$\therefore m(\widehat{AC}) + m(\widehat{CD}) + m(\widehat{DB}) = 180^\circ$

$\therefore m(\widehat{AC}) + m(\widehat{DB}) = 180^\circ - 70^\circ = 110^\circ$

Let $m(\widehat{AC}) = 5\,x$, $m(\widehat{DB}) = 6\,x$

$\therefore 5\,x + 6\,x = 110^\circ$ $\therefore 11\,x = 110^\circ$

$\therefore x = 10^\circ$

$\therefore m(\widehat{AC}) = 5 \times 10^\circ = 50^\circ$ (The req.)

[b] In Δ YZL : $\because YZ = LZ$

$\therefore m(\angle ZLY) = m(\angle ZYL) = 50^\circ$

$\therefore$ m ($\angle$ XZY) (tangency) = m ($\angle$ ZLY) (inscribed) $= 50°$

, $\because$ $\overrightarrow{XY}$, $\overrightarrow{XZ}$ are two tangents to the circle

$\therefore$ XZ = XY

$\therefore$ In Δ XYZ : m ($\angle$ XYZ) = m ($\angle$ XZY) = 50°

$\therefore$ m ($\angle$ X) = 180° − 2 × 50° = 80° (The req.)

4

[a] 1 $\because$ MA = 13 cm. = r

$\therefore$ L is a tangent to the circle M.

2 $\because$ MA = 10 cm. < r

$\therefore$ L is a secant to the circle M.

3 $\because$ MA = 15 cm. > r

$\therefore$ L is outside the circle M.

[b] $\because$ $\overline{XY}$ // $\overleftrightarrow{BD}$, $\overleftrightarrow{AB}$ is a transversal

$\therefore$ m ($\angle$ DBX) = m ($\angle$ BXY) (alternate angles)

, $\because$ m ($\angle$ C) (inscribed) = m ($\angle$ ABD) (tangency)

$\therefore$ m ($\angle$ C) = m ($\angle$ BXY)

$\therefore$ AXYC is a cyclic quadrilateral. (Q.E.D.)

5

[a] In Δ MCB : $\because$ MC = MB = r

$\therefore$ m ($\angle$ MCB) = m ($\angle$ MBC) = 45°

In Δ MAC : $\because$ MC = MA = r

$\therefore$ m ($\angle$ MCA) = m ($\angle$ MAC) = 25°

$\therefore$ m ($\angle$ ACB) = m ($\angle$ MCB) + m ($\angle$ MCA) = 70°

$\therefore$ m ($\angle$ AMB) = 2 m ($\angle$ ACB) = 140°

(central and inscribed angles subtended by $\overset{\frown}{AB}$)

(The req.)

[b] $\because$ $\overleftrightarrow{XY}$ // $\overline{BD}$ $\therefore$ m ($\overset{\frown}{BC}$) = m ($\overset{\frown}{DC}$)

$\therefore$ BC = DC

$\therefore$ Δ BDC is an isosceles triangle. (Q.E.D. 1)

$\therefore$ m ($\angle$ BDC) = m ($\angle$ CBD)

, $\because$ m ($\angle$ BDC) = m ($\angle$ BAC)

(two inscribed angles subtended by $\overset{\frown}{BC}$)

$\therefore$ m ($\angle$ CBD) = m ($\angle$ BAE)

$\therefore$ $\overleftrightarrow{BC}$ touches the circle passing through the vertices of Δ ABE. (Q.E.D. 2)

14 Qena

1

1 a 2 b 3 b 4 c 5 d 6 c

2

[a] $\because$ m ($\angle$ A) = $\frac{1}{2}$ [m ($\overset{\frown}{CE}$) − m ($\overset{\frown}{BD}$)]

$\therefore$ 36° = $\frac{1}{2}$ [104° − m ($\overset{\frown}{BD}$)]

$\therefore$ 72° = 104° − m ($\overset{\frown}{BD}$)

$\therefore$ m ($\overset{\frown}{BD}$) = 104° − 72° = 32° (First req.)

$\therefore$ m ($\overset{\frown}{DE}$) = m ($\overset{\frown}{BC}$) = $\frac{360° - (104° + 32°)}{2}$ = 112°

(Second req.)

[b] $\because$ $\overleftrightarrow{CB}$ is a tangent to the circle M at D

, $\overline{AD}$ is a diameter $\therefore$ $\overline{AD} \perp \overline{CB}$

In Δ ABC : $\because$ AB = AC , $\overline{AD} \perp \overline{CB}$

$\therefore$ D is the midpoint of $\overline{BC}$

$\therefore$ BC = 2 BD = 20 cm. , $\because$ m ($\angle$ B) = 60°

$\therefore$ Δ ABC is an equilateral triangle

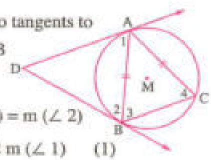
$\therefore$ CA = CB = AB = 20 cm.

$\therefore$ The perimeter of Δ ABC = 20 + 20 + 20

= 60 cm. (The req.)

3

[a] $\because$ $\overrightarrow{DA}$ and $\overrightarrow{DB}$ are two tangents to the circle M at A and B



$\therefore$ DA = DB

$\therefore$ In Δ ABD : m ($\angle$ 1) = m ($\angle$ 2)

$\therefore$ m ($\angle$ D) = 180° − 2 m ($\angle$ 1) (1)

In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ 3) = m ($\angle$ 4)

$\therefore$ m ($\angle$ BAC) = 180° − 2 m ($\angle$ 4) (2)

, $\because$ $\overline{AD}$ is a tangent-segment to the circle

$\therefore$ m ($\angle$ 4) (inscribed) = m ($\angle$ 1) (tangency) (3)

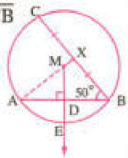
From (1) , (2) and (3) :

$\therefore$ m ($\angle$ BAC) = m ($\angle$ D)

$\therefore$ $\overleftrightarrow{AC}$ is a tangent to the circle passing through the vertices of Δ ABD (Q.E.D.)

[b] Const. : Draw $\overline{AM}$

Proof : $\because$ X is the midpoint of $\overline{CB}$



$\therefore \overline{MX} \perp \overline{BC}$

$\therefore m(\angle MXB) = 90°$

, $\because \overrightarrow{MD} \perp \overline{AB}$

$\therefore m(\angle MDB) = 90°$

From the quadrilateral BDMX :

$\therefore m(\angle DMX) = 360° - (90° + 90° + 50°)$

$= 130°$ (First req.)

, $\because \overline{MD} \perp \overline{AB}$

$\therefore$ D is the midpoint of $\overline{AB}$

$\therefore AD = \frac{1}{2} AB = 4$ cm.

In Δ ADM : $\because m(\angle ADM) = 90°$

, AM = r = 5 cm.

$\therefore MD = \sqrt{(AM)^2 - (AD)^2} = \sqrt{25 - 16} = \sqrt{9} = 3$ cm.

$\therefore DE = 5 - 3 = 2$ cm. (Second req.)

4

[a] In Δ ABE : $\because$ AB = BE

$\therefore m(\angle A) = m(\angle AEB)$

, $\because$ ABCD is a parallelogram

$\therefore m(\angle C) = m(\angle A)$

$\therefore m(\angle AEB) = m(\angle C)$

$\therefore$ BEDC is a cyclic quadrilateral. (Q.E.D.)

[b] $\because m(\angle BMC) = 2\, m(\angle BAC) = 2 \times 30° = 60°$

(central and inscribed angles subtended by $\widehat{BC}$)

, $\because$ MB = MC = r

$\therefore \Delta$ MBC is equilateral. $\therefore$ r = 7 cm.

$\therefore$ The area of the circle $= \pi \times r^2 = \frac{22}{7} \times (7)^2$

$= 154 \text{ cm}^2$ (The req.)

5

[a] $\because$ ABDC is a cyclic quadrilateral

$\therefore m(\angle ABD) + m(\angle ACD) = 180°$

$\therefore m(\angle ABD) = 180° - 115° = 65°$

, $\because \overline{AB}$ is a diameter $\therefore m(\angle ADB) = 90°$

$\therefore$ In Δ ABD : $m(\angle DAB) = 180° - (90° + 65°)$

$= 25°$ (The req.)

[b] $\therefore \overline{EB}$ and $\overline{ED}$ are two tangent-segments to the circle N $\therefore$ EB = ED (1)

$\therefore x + 3 = 7$ $\therefore x = 4$ cm.

, $\because \overline{EA}$ and $\overline{EC}$ are two tangent-segments to the circle M $\therefore$ EA = EC (2)

Subtracting (1) from (2) : $\therefore$ AB = CD

$\therefore 5 = y - 2$ $\therefore$ y = 7 cm. (The req.)

15 Matrouh

1

1 c	2 b	3 a	4 b	5 c	6 b

2

[a] $\because$ E is the midpoint of $\overline{AC}$

$\therefore \overline{ME} \perp \overline{AC}$ $\therefore m(\angle MEA) = 90°$

, $\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$ $\therefore m(\angle MDA) = 90°$

From the quadrilateral ADME :

$\therefore m(\angle DME) = 360° - (90° + 90° + 65°) = 115°$

(The req.)

[b] $\because \overline{AB}$ is a tangent-segment to the circle at B

$\therefore \overline{MB} \perp \overline{BA}$ $\therefore m(\angle B) = 90°$

$\therefore (MB)^2 = (MA)^2 - (AB)^2 = 100 - 64 = 36$

$\therefore MB = \sqrt{36} = 6$ cm.

$\therefore$ The area of $\Delta ABM = \frac{1}{2} \times AB \times BM$

$= \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$

(The req.)

3

[a] $\because m(\angle ACB) = \frac{1}{2} m(\angle AMB)$

(inscribed and central angles subtended by $\widehat{AB}$)

$\therefore m(\angle ACB) = \frac{1}{2} \times 120° = 60°$ (1)

, $\because \overrightarrow{CD} \parallel \overline{AB}$ $\therefore m(\widehat{CA}) = m(\widehat{CB})$

$\therefore$ CA = CB (2)

From (1) and (2) :

$\therefore \Delta$ CAB is an equilateral triangle. (Q.E.D.)

[b] $\because m(\widehat{AX}) = m(\widehat{AY})$

$\therefore m(\angle ACX) = m(\angle ABY)$

and they are drawn on $\overline{DE}$ and on one side of it

$\therefore$ BCED is a cyclic quadirlateral. (Q.E.D.)

4

[a] $\because$ ABCD is a cyclic quadrilateral

$\therefore m(\angle D) = m(\angle ABE) = 100°$

In Δ ACD :

$\therefore m(\angle DCA) = 180° - (100° + 40°) = 40°$

$\therefore m(\angle DCA) = m(\angle DAC) = 40°$

$\therefore AD = CD$

$\therefore \Delta$ ADC is an isosceles triangle. (Q.E.D.)

[b] $\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the circle.

$\therefore AB = AC$

In Δ ABC :

$\therefore m(\angle ACB) = m(\angle ABC) = \frac{180° - 40°}{2} = 70°$

$\therefore m(\angle D)$ (inscribed) $= m(\angle ACB)$ (tangency)

$= 70°$ (The req.)

5

[a] In Δ CMB :

$\because MC = MB = r$

$\therefore m(\angle MCB) = m(\angle MBC) = 40°$

$\therefore m(\angle CMB) = 180° - 2 \times 40° = 100°$

$\therefore m(\angle BAC) = \frac{1}{2} m(\angle CMB) = 50°$

(inscribed and central angles subtended by $\widehat{BC}$)

(The req.)

[b] $\because \overleftrightarrow{BX}$ is a tangent

$\therefore m(\angle CBX)$ (tangency) $= m(\angle CAB)$ (inscribed)

, $\because \overline{AD} \parallel \overleftrightarrow{BX}$, $\overleftrightarrow{DB}$ is a trasversal.

$\therefore m(\angle ADB) = m(\angle DBX)$ (alternate angles)

$\therefore m(\angle CAB) = m(\angle ADC)$

$\therefore \overline{AB}$ is a tangent-segment to the circle passing through the vertices of the triangle ACD (Q.E.D.)

Answers of examinations on Port Said specifications of Geometry

Exam 1 Port Said

First Answers of multiple choice questions

1 (b) 2 (c) 3 (c) 4 (a) 5 (d)
6 (c) 7 (b) 8 (d) 9 (c) 10 (c)
11 (d) 12 (b) 13 (a) 14 (b) 15 (b)
16 (a) 17 (d) 18 (b) 19 (d) 20 (a)
21 (c)

Second Answers of essay questions

22

$\because \overline{AB}$, $\overline{AC}$ are two tangent-segments to the smaller circle

$\therefore \overline{MX} \perp \overline{AB}$, $\overline{MY} \perp \overline{AC}$

, $\because MX = MY = r$
(lengths of two radii of the smaller circle)

$\therefore AB = AC$ (Q.E.D.)

23

$\because \Delta ABC$ is an equilateral triangle

$\therefore m(\angle B) = m(\angle BAC) = m(\angle BCA) = 60°$

$\therefore m(\angle B) + m(\angle D) = 60° + 120° = 180°$

$\therefore$ ABCD is a cyclic quadrilateral (Q.E.D.)

24

$\because m(\angle B) = \frac{1}{2} m(\widehat{BC}) = \frac{1}{2} \times 120° = 60°$ (1)

, $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents.

$\therefore AB = AC$ (2)

From (1) and (2) :

$\therefore \Delta ABC$ is an equilateral triangle.

$\therefore$ The perimeter of $\Delta ABC = 3 \times 5 = 15$ cm. (The req.)

Exam 2

First Answers of multiple choice questions

1 (c) 2 (a) 3 (a) 4 (b) 5 (b)
6 (b) 7 (d) 8 (c) 9 (c) 10 (a)
11 (b) 12 (a) 13 (a) 14 (d) 15 (a)
16 (a) 17 (a) 18 (c) 19 (d) 20 (c)
21 (b)

Second Answers of essay questions

22

$\because$ D is the midpoint of $\overline{AB}$

$\therefore \overline{MD} \perp \overline{AB}$ $\therefore m(\angle MDA) = 90°$

, $\because$ E is the midpoint of $\overline{AC}$

$\therefore \overline{ME} \perp \overline{AC}$ $\therefore m(\angle MEA) = 90°$

$\therefore$ From the quadrilateral ADME :

$m(\angle DME) = 360° - (90° + 90° + 60°) = 120°$

(The req.)

23

In ΔABD : $\because AB = AD$

$\therefore m(\angle ABD) = m(\angle ADB) = 40°$

$\therefore m(\angle A) = 180° - (40° + 40°) = 100°$

, $\because m(\angle A) + m(\angle BCD) = 100° + 80° = 180°$

$\therefore$ ABCD is a cyclic quadrilateral. (Q.E.D.)

24

$\because \overrightarrow{AB}$ is a tangent to the circle

$\therefore m(\angle ABC)$ (tangency) $= m(\angle BDC)$ (inscribed) $= 70°$

, $\because \overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle

$\therefore AB = AC$

In ΔABC : $\therefore m(\angle ACB) = m(\angle ABC) = 70°$

$\therefore m(\angle A) = 180° - (70° + 70°) = 40°$

(The req.)

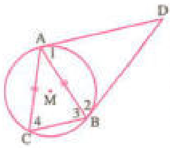
Exam 3

First Answers of multiple choice questions

1 (b) 2 (c) 3 (c) 4 (c) 5 (d)
6 (b) 7 (d) 8 (b) 9 (c) 10 (d)
11 (d) 12 (a) 13 (c) 14 (b) 15 (d)
16 (c) 17 (c) 18 (c) 19 (c) 20 (a)
21 (d)

Second Answers of essay questions

22



$\because$ $\overline{DA}$ and $\overline{DB}$ are two tangent-segments to the circle M at A and B

$\therefore$ DA = DB

$\therefore$ In Δ ABD : m ($\angle$ 1) = m ($\angle$ 2)

$\therefore$ m ($\angle$ D) = 180° – 2 m ($\angle$ 1) (1)

In Δ ABC : $\because$ AB = AC

$\therefore$ m ($\angle$ 3) = m ($\angle$ 4)

$\therefore$ m ($\angle$ BAC) = 180° – 2 m ($\angle$ 4) (2)

$\because$ $\overline{AD}$ is a tangent-segment to the circle

$\therefore$ m ($\angle$ 4) (inscribed) = m ($\angle$ 1) (tangency) (3)

From (1) , (2) and (3) : $\therefore$ m ($\angle$ D) = m ($\angle$ BAC)

$\therefore$ $\overleftrightarrow{AC}$ is a tangent to the circle passing through the vertices of Δ ABD (Q.E.D.)

23

$\because$ MH = ME = r (lengths of two radii of the circle)

, $\because$ HX = EY

By subtracting $\therefore$ MX = MY

, $\because$ X is the midpoint of $\overline{AC}$ $\therefore$ $\overline{MX} \perp \overline{AC}$

, $\because$ Y is the midpoint of $\overline{BC}$ $\therefore$ $\overline{MY} \perp \overline{BC}$

$\therefore$ AC = BC , $\because$ m ($\angle$ A) = 60°

$\therefore$ Δ ABC is an equilateral triangle (Q.E.D.)

24

$\because$ ABCD is a cyclic quadrilateral

$\therefore$ m ($\angle$ D) = m ($\angle$ ABE) = 100°

In Δ ACD :

$\therefore$ m ($\angle$ ACD) = 180° – (100° + 40°) = 40°

$\therefore$ m ($\angle$ CAD) = m ($\angle$ ACD) = 40°

$\therefore$ m $(\widehat{CD})$ = m $(\widehat{AD})$ (Q.E.D.)

Exam 4

First Answers of multiple choice questions

1 (a)	2 (d)	3 (b)	4 (c)	5 (b)
6 (b)	7 (a)	8 (c)	9 (d)	10 (d)
11 (b)	12 (c)	13 (d)	14 (a)	15 (d)
16 (b)	17 (c)	18 (c)	19 (d)	20 (d)
21 (c)				

Second Answers of essay questions

22

$\because$ $\overline{AD}$, $\overline{AF}$ are two tangent-segments to the circle.

$\therefore$ AD = AF = 5 cm.

, $\because$ $\overline{BD}$, $\overline{BE}$ are two tangent-segments to the circle

$\therefore$ BD = BE = 4 cm.

, $\because$ $\overline{CE}$, $\overline{CF}$ are two tangent-segments to the circle

$\therefore$ CE = CF = 3 cm.

$\therefore$ The perimeter of Δ ABC = 5 + 5 + 4 + 4 + 3 + 3

= 24 cm. (The req.)

23

$\because$ $\overline{AB}$ is a diameter $\therefore$ m ($\angle$ ACB) = 90°

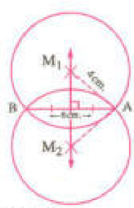
, $\because$ $\overrightarrow{DE} \perp \overrightarrow{AB}$ $\therefore$ m ($\angle$ ADE) = 90°

, $\because$ m ($\angle$ ACE) = m ($\angle$ ADE) = 90°

and they are drawn on $\overline{AE}$ and on one side of it

$\therefore$ ACDE is a cyclic quadrilateral (Q.E.D.)

24



We can draw two circles.

Exam 5

First Answers of multiple choice questions

1 (c)	2 (a)	3 (b)	4 (c)	5 (b)
6 (c)	7 (b)	8 (c)	9 (b)	10 (a)
11 (a)	12 (d)	13 (d)	14 (b)	15 (a)
16 (d)	17 (a)	18 (c)	19 (a)	20 (c)
21 (d)				

Second Answers of essay questions

22

$\because$ $\overline{DC}$ is a diameter

$\therefore$ m ($\angle$ DBC) = 90°

$\therefore$ m ($\angle$ ABC) = 135° – 90° = 45°

, $\because$ $\overrightarrow{BA}$ is a tangent to the circle M at B

$\therefore$ m ($\angle$ D) (inscribed) = m ($\angle$ ABC) (tangency) = 45°

$\therefore$ In Δ BCD : m ($\angle$ C) = 180° – (90° + 45) = 45°

$\therefore$ m ($\angle$ C) = m ($\angle$ ABC) = 45° and they are alternate angles.

$\therefore$ $\overline{DC}$ // $\overrightarrow{BA}$ (Q.E.D.)

23

$\because$ $\overleftrightarrow{MN}$ is the line of centres

, $\overline{AB}$ is the common chord

$\therefore$ $\overleftrightarrow{MN} \perp \overline{AB}$

$\therefore$ m ($\angle$ AYN) = 90°

, $\because$ X is the midpoint of $\overline{AC}$

$\therefore$ $\overline{NX} \perp \overline{AC}$ $\therefore$ m ($\angle$ AXN) = 90°

From the quadrilateral AXNY

$\therefore$ m ($\angle$ BAC) = 360° – (90° + 90° + 80°) = 100°

(The req.)

24

$\because$ E is the midpoint of $\overline{AD}$

$\therefore$ $\overline{ME} \perp \overline{AD}$ $\therefore$ m ($\angle$ MEC) = 90°

, $\because$ $\overline{BC}$ is a tangent-segment to the circle at B

$\therefore$ $\overline{MB} \perp \overline{BC}$ $\therefore$ m ($\angle$ MBC) = 90°

, $\because$ m ($\angle$ MEC) + m ($\angle$ MBC) = 90° + 90° = 180°

$\therefore$ The figure BMEC is a cyclic quadrilateral (Q.E.D.)

كيفية طباعة صفحات معينة من ملف معين

مثلا ازاى نطبع الصفحات من صفحة 4 الى صفحة 9